TOPIC 5 FORMULA SHEET

5.1 PERPENDICULAR & ANGLE BISECTORS

Segment Bisector formula: $(x_1, y_1)\ (x_2, y_2) = \left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$

Equidistant – a point that is the same distance from 2 points is equidistant from the points.

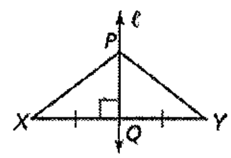
THEOREM 5-1

Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

PROOF: SEE EXAMPLE 2.

If...



Then... $PX = PY$

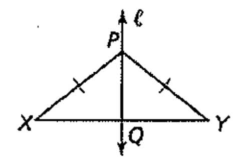
THEOREM 5-2

Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

PROOF: SEE EXAMPLE 2 TRY IT.

If...



Then... $XQ = YQ$ and $\overleftrightarrow{PQ} \perp \overline{XY}$

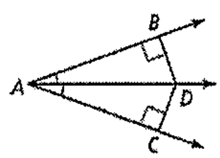
THEOREM 5-3

Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

PROOF: SEE EXERCISE 9.

If...



Then... $BD = CD$

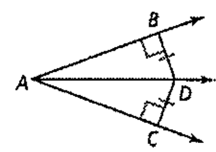
THEOREM 5-4

Converse of the Angle Bisector Theorem

If a point is equidistant from two sides of an angle, then it is on the angle bisector.

PROOF: SEE EXERCISE 10.

If...



Then... $m\angle BAD = m\angle CAD$

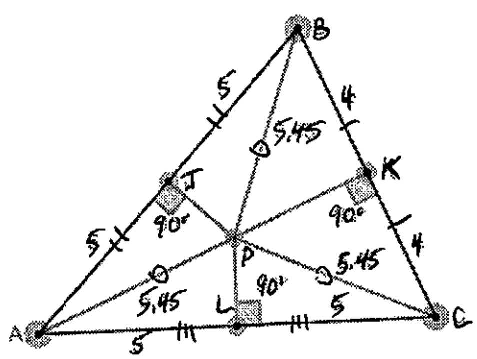
5.2 Bisectors in Triangles

THEOREM 5-5

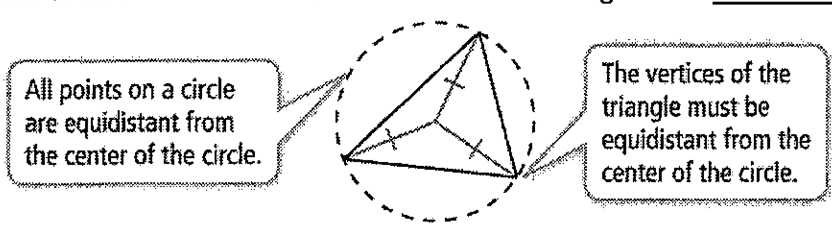
Concurrency of Perpendicular Bisectors

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

The perpendicular bisectors of the sides of the triangle intersect at P, and $PA = PB = PC$.



The circle that contains all three vertices of a triangle is the ***circumscribed*** circle of the triangle.



Acute inside
Rt on
Obtuse outside

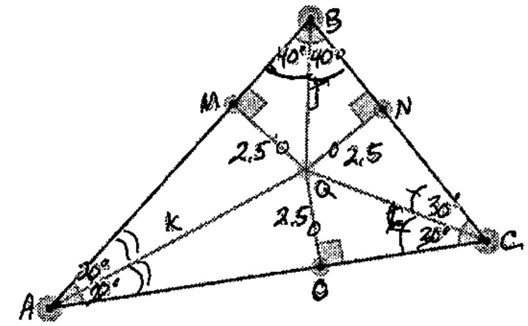
The point of concurrency of the perpendicular bisectors of a triangle is the ***circumcenter***, so the circumcenter is the center of the circumscribed circle of the triangle.

THEOREM 5-6

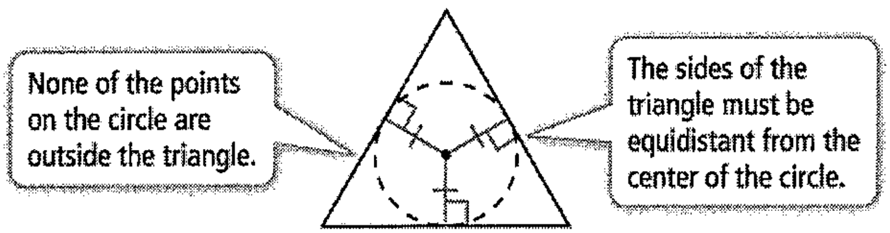
Concurrency of Angle Bisectors

The angle bisectors of the angles of a triangle are concurrent at a point equidistant from the sides of the triangle.

The bisectors of the angles of the triangle intersect at Q, and $QM = QN = QO$.



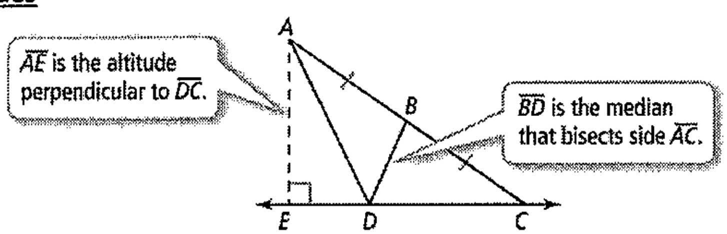
The circle that intersects each side of a triangle at exactly one point and has no points outside of the triangle is the ***inscribed*** circle of the triangle.



always inside

The point of concurrency of the angle bisectors of a triangle is the ***incenter***, so the ***incenter*** is the center of the inscribed circle of the triangle.

5.3 Medians & Altitudes



An ***altitude*** is a perpendicular segment from a vertex of a triangle to the line containing the side opposite the vertex.

A ***median*** of a triangle is a segment that has endpoints at a vertex and the midpoint of the side opposite the vertex.

$\overline{AE}$ is the altitude perpendicular to $\overline{DC}$.
$\overline{BD}$ is the median that bisects side $\overline{AC}$.

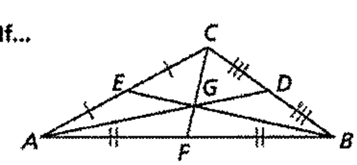
THEOREM 5-7

Concurrency of Medians

The medians of a triangle are concurrent at a point that is two-thirds the distance from each vertex to the midpoint of the opposite side.

PROOF: SEE LESSON 9-2.

If...



Then...

$AG = \frac{2}{3}AD \quad BG = \frac{2}{3}BE \quad CG = \frac{2}{3}CF$

The point of concurrency of the medians of a triangle is called the ***centroid***. G is centroid

always inside

THEOREM 5-8

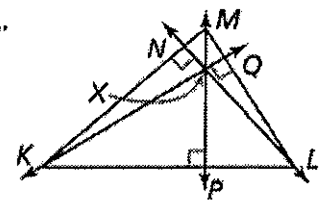
The ***orthocenter*** is the point of concurrency of the altitudes of a triangle. X is orthocenter.

Concurrency of Altitudes

The lines that contain the altitudes of a triangle are concurrent.

PROOF: SEE LESSON 9-2.

If...



Then... $\overleftrightarrow{KQ}$, $\overleftrightarrow{LN}$, and $\overleftrightarrow{MP}$ are concurrent at X

acute inside
rt on
obtuse outside

5.4 Inequalities in One Triangle

THEOREM 5-9

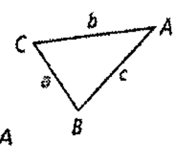
The angles of a triangle add up to 180°

THEOREM 5-9

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

PROOF: SEE EXERCISE 13.

If... $b > a$



Then... $m\angle B > m\angle A$

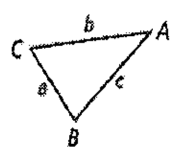
THEOREM 5-10

Converse of Theorem 5-9

If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.

PROOF: SEE EXAMPLE 3.

If... $m\angle B > m\angle A$



Then... $b > a$

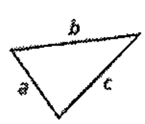
THEOREM 5-11

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

PROOF: SEE EXERCISE 14.

If...



Then... $a + b > c$
$a + c > b$
$b + c > a$

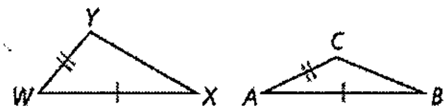
THEOREM 5-12

Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the included angles are not congruent, then the longer third side is opposite the larger included angle.

PROOF: SEE EXERCISE 9.

If... $m\angle YWX > m\angle CAB$



Then... $XY > BC$

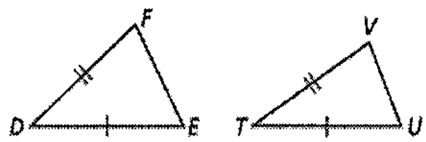
THEOREM 5-13

Converse of the Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the third sides are not congruent, then the larger included angle is opposite the longer third side.

PROOF: SEE EXAMPLE 3.

If... $EF > UV$



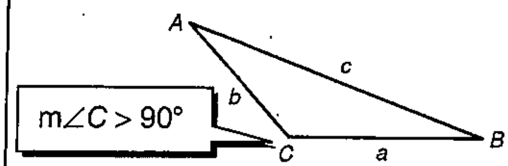
Then... $m\angle D > m\angle T$

A **Pythagorean triple** is a set of three nonzero whole numbers *a*, *b*, and *c* that satisfy the equation $a^2 + b^2 = c^2$.

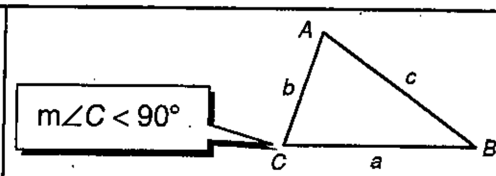
Pythagorean Triples	Not Pythagorean Triples
3, 4, 5, 5, 12, 13	2, 3, 4 6, 9, $\sqrt{117}$

You can use the following theorem to classify triangles by their angles if you know their side lengths. Always use the length of the longest side for *c*.

Pythagorean Inequalities Theorem



If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.

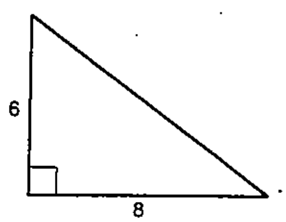


If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.

Consider the measures 2, 5, and 6. They can be the side lengths of a triangle since $2 + 5 > 6$, $2 + 6 > 5$, and $5 + 6 > 2$. If you substitute the values into $c^2 < a^2 + b^2$, you get $36 > 29$. Since $c^2 > a^2 + b^2$, a triangle with side lengths 2, 5, and 6 must be obtuse.

Find the missing side length. Tell whether the side lengths form a Pythagorean triple. Explain.

5.



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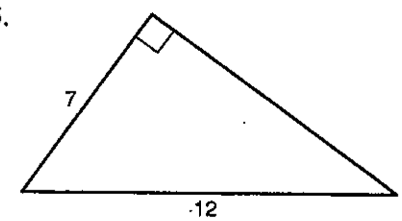
6.



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Tell whether the measures can be the side lengths of a triangle. If so, classify the triangle as acute, obtuse, or right.

7. 4, 7, 9 ______________________

8. 10, 13, 16 ______________________

9. 8, 8, 11 ______________________

10. 9, 12, 15 ______________________

11. 5, 14, 20 ______________________

12. 4.5, 6, 10.2 ______________________

The Triangle Inequality Theorem describes a relationship among the lengths of the sides of a triangle. The following two theorems relate the lengths of the sides to the measures of the angles.

UNEQUAL SIDES THEOREM

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

UNEQUAL ANGLES THEOREM

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

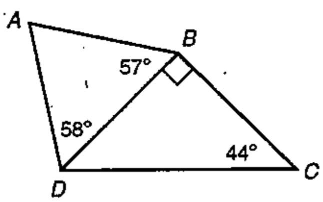
1. In ΔXYZ, $XY = 9.3$, $YZ = 7.6$, and $XZ = 8.05$. Name the largest and smallest angles of ΔXYZ.

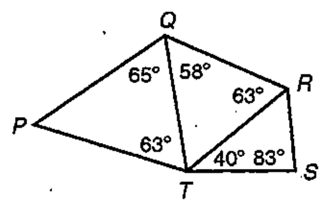
____

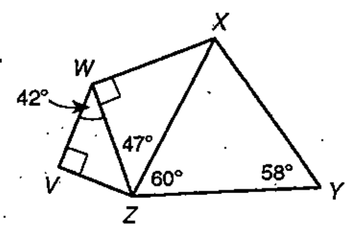
2. In ΔJKL, $m\angle J = 62°$ and $m\angle K = 57°$. Name the longest and shortest sides of ΔJKL.

____

In each figure, list the segments in order from longest to shortest.

3. 

4. 

5. 

______________ ______________ ______________

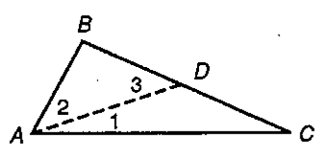
______________ ______________ ______________

On a separate sheet of paper, write a proof of each theorem.

6. *Unequal Sides Theorem*

Given: ΔABC with $BC > AB$

Prove: $m\angle BAC > m\angle C$.

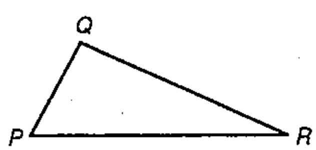


Plan for proof: Locate point D on $\overline{BC}$ such that $BD = BA$. Draw $\overline{AD}$. Explain why $m\angle BAC > m\angle 3$, $m\angle 3 > m\angle C$, and so $m\angle BAC > m\angle C$.

7. *Unequal Angles Theorem*

Given: ΔPQR with $m\angle P > m\angle R$

Prove: $QR > QP$



Plan for proof: The three possible relationships between QR and QP are $QR = QP$, $QR < QP$, and $QR > QP$. Show that the first two relationships listed are impossible.

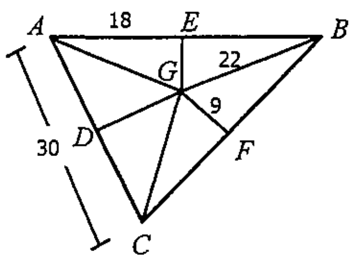
4/1

Group Members: **Block:** ________

Centers of Triangles Review

Directions: Work together to complete each problem. Do not divide up the work! Each person should be participating. At the end of the block, one person's paper will be chosen at random to be graded for the group.

1. If G is the circumcenter of ΔABC, find each missing measure.



a) AD = ________

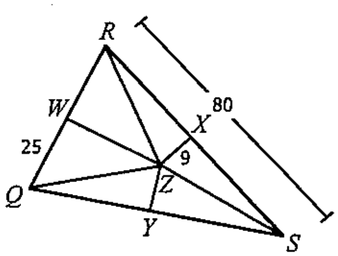
b) FC = ________

c) EB = ________

d) AG = ________

e) EG = ________

2. If Z is the circumcenter of ΔQRS, find each missing measure.



a) QR = ________

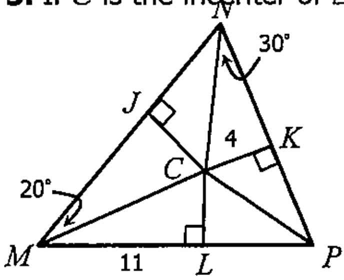
b) RZ = ________

c) XS = ________

d) ZS = ________

e) WZ = ________

3. If C is the incenter of ΔMNP, find each missing measure.



a) $m\angle CML$ = ________

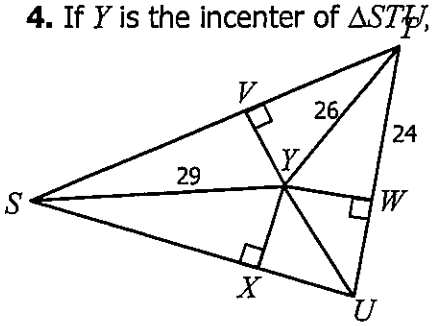
b) $m\angle MNP$ = ________

c) $m\angle NPC$ = ________

d) JC = ________

e) MC = ________

4. If Y is the incenter of ΔSTU, find each missing measure.



a) VT = ________

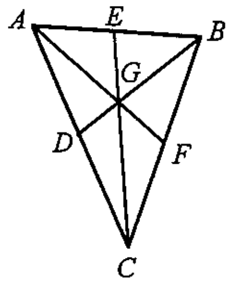
b) YW = ________

c) SX = ________

d) YX = ________

e) SV = ________

5. If G is the centroid of $\triangle ACE$, $AG = 26$, $BC = 44$, and $DG = 12$, find each missing measure.



a) $GF =$ ______

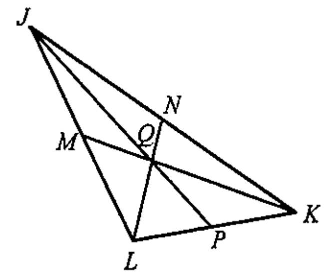
b) $AF =$ ______

c) $FC =$ ______

d) $GB =$ ______

e) $DB =$ ______

6. If Q is the centroid of $\triangle JKL$, $LN = 72$, $JP = 93$, and $MK = 78$, find each missing measure.



a) $LQ =$ ______

b) $QN =$ ______

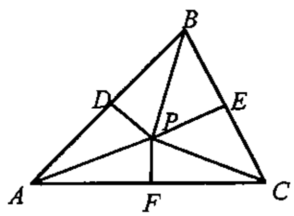
c) $QP =$ ______

d) $JQ =$ ______

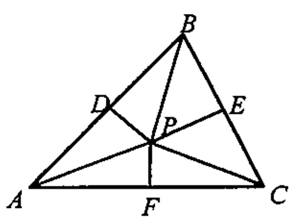
e) $QK =$ ______

For questions 7 and 8, P is the circumcenter of $\triangle ABC$.

7. If $BE = 8x - 11$, and $EC = 13x - 31$, find BC.

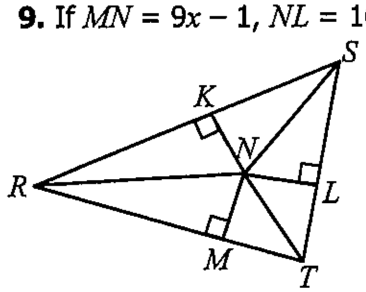


8. If $BP = 9x - 29$, $AP = 5x - 1$, and $PF = 15$, find FC.

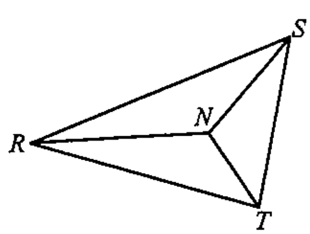


For questions 9 and 10, N is the incenter of $\triangle RST$.

9. If $MN = 9x - 1$, $NL = 16x - 15$, find KN.



10. If $m\angle RST = 3x + 17$, $m\angle STR = 8x - 32$, and $m\angle TRS = 2x$, find $m\angle RSN$.



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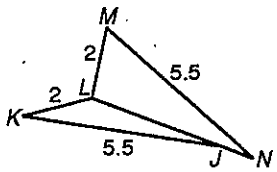
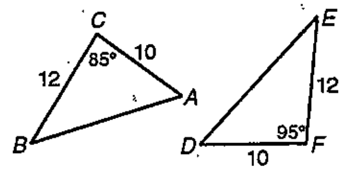
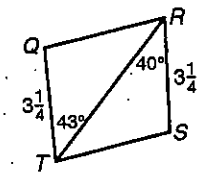
Compare the given measures.







1. m∠K and m∠M

______

2. AB and DE

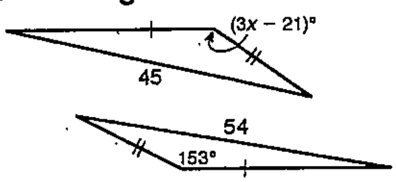
______

3. QR and ST

______

Find the range of values for x.

4.



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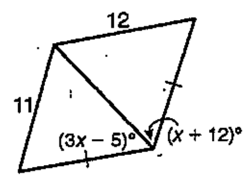

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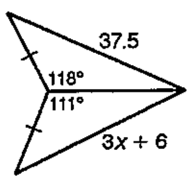

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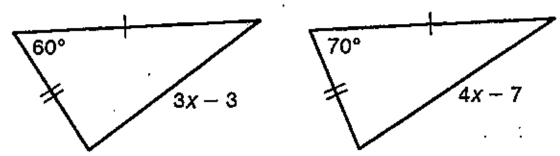
7.



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8. You have used a compass to copy and bisect segments and angles and to draw arcs and circles. A compass has a drawing leg, a pivot leg, and a hinge at the angle between the legs. Explain why and how the measure of the angle at the hinge changes if you draw two circles with different diameters.

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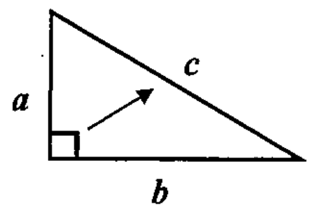
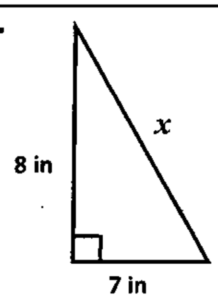
Name:	Class:
Topic:	Date:

Main Ideas/Questions	Notes
Parts of a Right Triangle	 ▪ Sides *a* and *b* are called __________ ▪ Side *c* is called the ______________
What is the Pythagorean Theorem?	The **Pythagorean Theorem** is used to find a **missing side length** on a ____________ triangle! **Formula:**

Practice! Find the missing side of each triangle. Round to the nearest tenth.

1.



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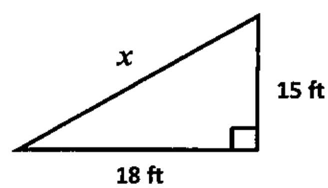

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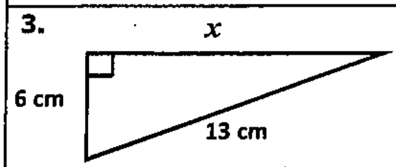
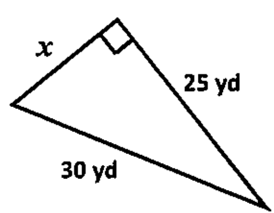


4.



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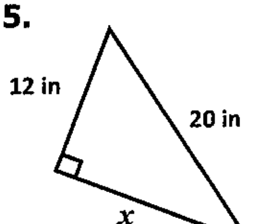

6.