Name Date

Ratios and rates

A ratio is a comparison by division of two numbers with the same units. Ratios are often expressed as fractions in simplest form or as decimals. A ratio can also be expressed with a colon (1:3, or one to three). For example, express the following as a fraction in simplest form and as a decimal:

3 children out of 9 children have a sibling.

$\frac{3}{9} = \frac{1}{3}$ Fraction in simplest form

$\frac{3}{9} = \frac{1}{3} = 0.3\overline{3}$ Decimal with a line over the 3 to show repeating

Thus, $\frac{1}{3}$ or $0.3\overline{3}$, of the children have a sibling.

A rate is a comparison of two measurements with different units of measure and is considered to be a special kind of ratio. For example, the ratio of 1 dozen golf balls for $6 can be written: $\frac{1 \text{ doz. golf balls}}{\$6}$. This rate compares golf balls and their price.

To find the price per golf ball, first simplify the rate so the denominator is $1. A rate with a denominator of 1 is called a unit rate.

What is the cost of one golf ball using the above information?

$\frac{1 \text{ doz. golf balls}}{\$6}$ Set up the initial rate.

$\frac{12 \text{ golf balls}}{\$6}$ Simplify 1 doz. = 12 golf balls.

$\frac{12 \div 6}{6 \div 6} = \frac{2}{1}$ To get the unit rate, divide numerator and denominator by 6. Notice 1 is now in the denominator.

$\frac{2 \text{ golf balls}}{\$1}$ Replace in problem adding the units.

Since 2 golf balls cost $1, 1 golf ball costs $.50.

1. What is the difference between a rate and a ratio? Give an example of each.

2. Write a ratio that compares the boys to girls in your class.

Express each ratio or rate as a fraction in simplest form.

3. 12 out of 144

4. 7 out of 9 apples

5. 51 : 17

6. 72 to 9

Express each ratio as a unit rate.

7. 210.8 miles on 12.4 gallons

8. 2.5 in. of rain in 10 hours

9. $10.20 for 15 lbs.

10. $62.50 for 25 tickets

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Equivalent ratios

To find equivalent ratios, simply multiply the numerator and the denominator by the same nonzero number.

$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

Find a ratio that is equivalent to $\frac{3}{4}$.

Choose a number to multiply both the numerator and the denominator by. In this case, 2 was chosen.

To change a ratio to lowest terms, or simplified form, simply divide the numerator and the denominator by the same nonzero number.

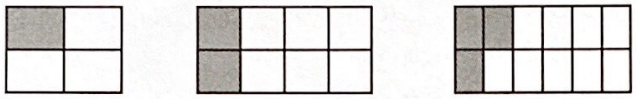
$\frac{8}{14} = \frac{8 \div 2}{14 \div 2} = \frac{4}{7}$

Change $\frac{8}{14}$ to lowest terms.

The GCF of 8 and 14 is 2 so divide the numerator and the denominator both by 2.

Note: To know when a ratio is in its simplest form, look to make sure the numerator and the denominator have no common whole number factors other than 1.

1. Write a ratio of shaded regions to unshaded regions in each rectangle. State if these ratios are equivalent.



Find two equivalent ratios for each ratio.

2. $\frac{2}{3}$

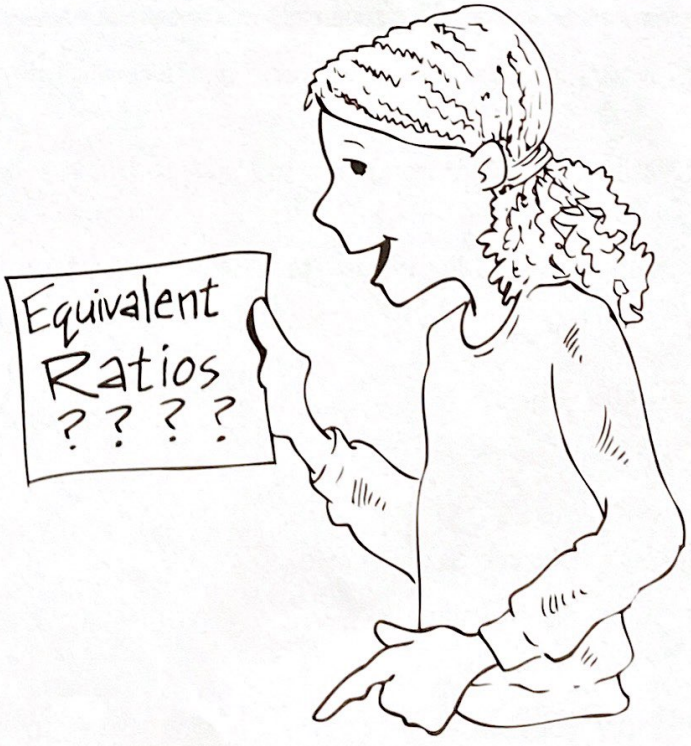
3. $\frac{7}{9}$

4. $\frac{1}{8}$

5. $\frac{13}{20}$

6. $\frac{3}{5}$

7. $\frac{10}{17}$



Write each ratio in lowest terms.

8. $\frac{2}{10}$

9. $\frac{14}{56}$

10. $\frac{14}{21}$

11. $\frac{32}{64}$

12. $\frac{55}{33}$

13. $\frac{25}{65}$

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Proportions

A proportion is an equation that names two equivalent ratios.

$\frac{3}{4} = \frac{12}{16}$ This is a proportion.

In this proportion, 3 and 16 are called the extremes, and 4 and 12 are called the means. 3×16 and 4×12 are called the cross products. In a proportion, the cross products are always equal, meaning the product of the extremes is equal to the product of the means. For example, $3 \times 16 = 48$ and $4 \times 12 = 48$. This is a great test to show a true proportion. Look at the following examples:

Use cross products to determine if each pair of ratios forms a proportion.

1. $\frac{5}{6}, \frac{10}{14}$

$\frac{5}{6} = \frac{10}{14}$

$5 \cdot 14 = 6 \cdot 10$

$70 \neq 60$

Thus, $\frac{5}{6} \neq \frac{10}{14}$ is not a proportion.

2. $\frac{1}{4}, \frac{8}{32}$

$\frac{1}{4} = \frac{8}{32}$

$1 \cdot 32 = 4 \cdot 8$

$32 = 32$

Thus, $\frac{1}{4} = \frac{8}{32}$ forms a proportion.

Solve the proportion $\frac{n}{3} = \frac{6}{9}$.

$\frac{n}{3} = \frac{6}{9}$

$n \cdot 9 = 3 \cdot 6$ Multiply cross products.

$\frac{9n}{9} = \frac{18}{9}$ Divide both sides by 9.

$n = 2$ Thus, the solution is 2.

1. What is the relationship of ratios and proportions? Give an example of a proportion.

2. State the steps to use to solve the proportion $\frac{2}{x} = \frac{20}{30}$ and solve. Identify the means and the extremes.

Use cross products to tell whether each sentence is true. Write **T** (true) or **F** (false).

3. $\frac{3}{2} = \frac{63}{42}$

4. $\frac{2}{5} = \frac{14}{35}$

5. $\frac{10}{16} = \frac{5}{9}$

6. $\frac{7}{5} = \frac{36}{20}$

Solve each proportion.

7. $\frac{3}{4} = \frac{n}{16}$

8. $\frac{46}{92} = \frac{n}{100}$

9. $\frac{2}{3} = \frac{24}{n}$

10. $\frac{n}{2} = \frac{56}{112}$

Set up a proportion to use to solve for each variable and solve.

11. 9 gallons for \$27
x gallons for \$9.60

12. 25 candies per 5 boxes
150 candies per *x* boxes

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Using proportions to solve problems

Ratios, Proportions, and Percents

The most important point to remember when using proportions to help solve problems is to be sure to have the same unit in both numerators, as well as the same unit in both denominators. Once the proportion is set up, simply solve as usual. Look at the following examples:

1. Out of 10 girls, 4 were chosen to go to the state competition. At this rate, how many girls would be chosen out of 50?

$\frac{4}{10} = \frac{x}{50}$ $\frac{\text{girls in competition}}{\text{total girls}}$ Set up initial proportion. Be sure units match in the numerators and the units match in the denominators.

$4 \cdot 50 = 10 \cdot x$ Multiply cross products.

$\frac{200}{10} = \frac{10x}{10}$ Divide both sides by 10.

$x = 20$ Thus, 20 girls would be chosen out of 50.

2. If a 9-lb. turkey takes 180 minutes to cook, how long would a 6-lb. turkey take to cook?

$\frac{9}{180} = \frac{6}{x}$ $\frac{\text{lb.}}{\text{minutes}}$ Set up initial proportion.

$9 \cdot x = 6 \cdot 180$ Multiply cross products.

$\frac{9x}{9} = \frac{1080}{9}$ Divide both sides by 9.

$x = 120$ Thus, it would take a 6-lb. turkey 120 minutes to cook.

Set up a proportion to represent each problem and solve.

1. There are 220 calories in 4 ounces of beef. How many calories are there in 5 ounces?

2. If John can buy 8 liters of soft drinks at the store for $6.40, how much does it cost him to buy 12 liters?

3. Sherri bought a package of pens that contained 15 pens. How many packages should she buy if she needs 240 pens?

4. Steve won his election by a margin of 7 to 2. His opponent had 3,492 votes. How many votes did Steve have?

5. A car traveled 325 miles in 5 hours. How far did the car travel in 9 hours?

6. A recipe asks for 1 ½ cups of chocolate chips for 60 cookies. How many cups would be needed for 36 cookies?

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Meaning of percent

Percent is a ratio that compares a number to 100. The symbol for percent is %. For example, $\frac{5}{100}$ is 5% and $\frac{23}{100}$ is 23%. Any percent can be changed to a fraction or a decimal. When percents are changed to fractions, the fractions should be simplified when possible.

1. Change 65% to a fraction and then to a decimal.

$65\% = \frac{65}{100}$ Place 65 over 100. $65\% = 0.65$

$= \frac{13}{20}$ Simplify fraction. To change to a decimal, simply move decimal two places to the left (÷ 100).

2. Change 150% to a fraction and then to a decimal.

$150\% = \frac{150}{100}$ Place 150 over 100. $150\% = 1.5$

$= \frac{3}{2} = 1\frac{1}{2}$ Simplify fraction. Move decimal two places left.

3. Change $62\frac{1}{2}\%$ to a fraction and then to a decimal.

$62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100}$ Place $62\frac{1}{2}$ over 100.

$= 62\frac{1}{2} \div 100$ Divide by 100.

$= \frac{125}{2} \cdot \frac{1}{100}$ Change to multiplication problem.

$= \frac{125}{200} = \frac{5}{8}$ Simplify fraction.

$62\frac{1}{2}\% = 62.5\% = 0.625$ Change $\frac{1}{2}$ to .5 and move decimal two places left.

Write each of the following as a percent.

1. 60 out of 100 **2.** 55 to 100 **3.** $\frac{n}{100}$

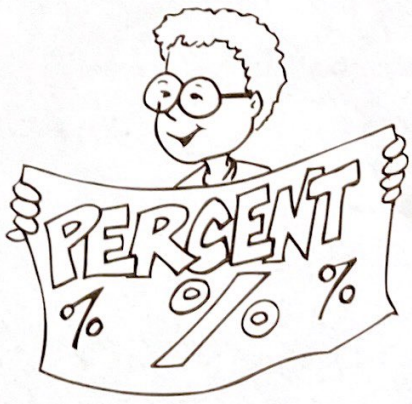
4. 49 out of 100 **5.** $\frac{25}{100}$ **6.** $\frac{125}{100}$

Change each percent to a fraction in lowest terms and then to a decimal.

7. 5% **8.** $28\frac{1}{2}\%$ **9.** 8.75% **10.** 150% **11.** 60%

Complete the table.

	percent	fraction	decimal
12.	$62\frac{1}{2}\%$		
13.	48%		
14.	0.3%		
15.	$\frac{3}{5}\%$		



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Changing numbers to percents

To change a decimal to a percent, move the decimal point two places to the right and write the symbol %.

Change 0.765 to a percent.

$0.765 = 76.5\%$ Move decimal two places to the right and write %.

To change a fraction to a percent, set up a percent proportion and then solve, placing the percent symbol after the solution.

Change $\frac{3}{4}$ to a percent.

$\frac{3}{4} = \frac{n}{100}$ Set up percent proportion by setting fraction equal to a number over 100. Solve as a proportion.

$3 \times 100 = 4 \times n$ Multiply cross products.

$\frac{300}{4} = \frac{4n}{4}$ Divide both sides by 4.

$75\% = n$ Solve for unknown and place % after number.

Thus, $\frac{3}{4}$ is equal to 75 percent.

1. Describe how to change a decimal to a percent. Give an example.

2. Describe how to change a fraction to a percent. Give an example.

Change each of the following to a percent.

3. 0.32

4. 0.125

5. $\frac{1}{3}$

6. $\frac{4}{25}$

7. 3.5

8. 1.475

9. $\frac{3}{5}$

10. $3\frac{2}{25}$

11. The interest for a home loan is stated as \$7.25 per \$100. What is the percent of interest?

Change each ratio to a percent.

12. 4 to 2

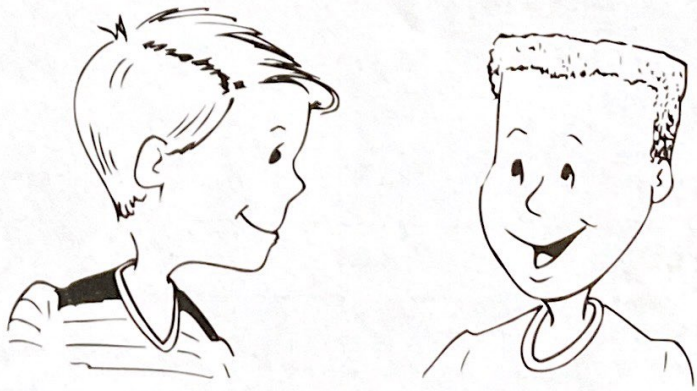
13. $1\frac{1}{2}$ to $1\frac{1}{4}$

14. $\frac{1}{2}$ to $\frac{3}{4}$

15. 1.2 : 2.4

16. 5 : 6

17. 6 : 4



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Three kinds of percent problems

When using proportions to solve percent problems, the denominator of one of the fractions is always going to be 100. Any of the other three numbers can be unknown and asked to be solved. Three problems are presented below in which a different unknown is solved for in each problem.

1. 15 is what percent of 75?

$\frac{15}{75} = \frac{n}{100}$	Identify what is being asked for: the percent. Set up a proportion.
$15 \cdot 100 = 75 \cdot n$	Multiply cross products.
$\frac{1,500}{75} = \frac{75n}{75}$	Divide both sides by 75.
$20 = n$	Solve for the unknown. Thus, 15 is 20% of 75.

2. 9 is 30% of what number?

$\frac{9}{n} = \frac{30}{100}$	Identify what is being asked for: denominator of the fraction.
$9 \cdot 100 = 30 \cdot n$	Multiply cross products.
$\frac{900}{30} = \frac{30n}{30}$	Divide both sides by 30.
$30 = n$	Solve for the unknown. Thus, 9 is 30% of 30.

3. What is 85% of 120?

$\frac{n}{120} = \frac{85}{100}$	Identify what is being asked for: numerator of the fraction.
$\frac{100n}{100} = \frac{85 \times 120}{100}$	Multiply cross products. Divide both sides by 100.
$n = 102$	Solve for the unknown. Thus, 102 is 85% of 120.

Note: In example 3, it would be just as easy to change the % to a decimal and multiply by 120, because "of" in mathematics simply means multiply.

Set up a proportion to represent each problem. Solve the proportion.

1. 45 is what percent of 90?

2. What percent of 100 is 19?

3. What is 75% of 60?

4. 35% is 7 out of what?

5. 62% of what number is 9.3?

6. 60% of what number is 50.4?

7. 7 out of 28 is what percent?

8. 90 is 100% of what number?

9. How much is 72% of 54?

10. What percent of 132 is 76.56?

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Solving percent word problems

Just as word problems are solved through mathematics, the same methods can be used to solve those problems involving percents.

1. Bob must make at least 75% of his free throws to advance to the final competition. How many free throws must Bob make out of 52 to advance?

First, identify what you are looking for: # of free throws.

$\frac{n}{52} = \frac{75}{100}$ Set up a proportion with given information and solve.

$n \times 100 = 52 \times 75$ Multiply cross products.

$\frac{100n}{100} = \frac{3,900}{100}$ Divide both sides by 100.

$n = 39$ Solve for the unknown. Thus, Bob must make 39 free throws.

2. Each item at a sale was reduced 25%. What was the regular price of a shirt that is reduced $9?

First, identify what you are looking for: item's regular price.

$\frac{9}{n} = \frac{25}{100}$ Set up a proportion with given information and solve.

$\frac{900}{25} = \frac{25n}{25}$ Multiply cross products. Divide both sides by 25.

$36 = n$ Solve for the unknown. Thus, the regular price of the shirt is $36.

Solve each problem by setting up a proportion.

1. Molly made $30 in tips from her customers. If the total of her customers' bills was $200, what percent did her customers tip?

2. How many problems did Robert get right out of 40 if he received an 87.5% on his test?

3. Mary has sold 90 boxes of cookies. If her goal was to sell 120 boxes, what percentage of her goal has she sold?

4. How much did Tom pay in income tax on a gross income of $50,000 if 9% of his income was taxed?

5. How much is a $48 shirt that is on sale for 25% off?

6. Janet borrowed $5,500 from the bank at an interest rate of $7\frac{1}{2}\%$ for one year. Assuming she pays it back on time, how much interest will she pay?

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Percent of change

Ratios, Proportions, and Percents

The percent of change is the ratio of the amount of change to the original amount. When an amount increases, the percent of change is the percent of increase. When the amount decreases, the percent of change is the percent of decrease, which is negative.

1. Find the percent of change, or the percent of increase, from $150 to $175.

175 − 150 = 25	First, subtract to find the amount of change.
25 ÷ 150 = 0.167	Divide amount of change by original amount.
0.167 = 16.7%	Change decimal to a percent.

Thus, the percent of increase is 16.7%.

2. Find the percent of change, or the percent of decrease, from 145 lbs. to 125 lbs.

145 − 125 = 20	First, subtract to find the amount of change.
20 ÷ 145 = 0.138	Divide amount of change by original amount.
0.138 = 13.8%	Change decimal to a percent.

Thus, the percent of decrease is 13.8%.

State whether each of the following is a percent of increase or a percent of decrease. Then find the actual percent of increase or decrease. Round to the nearest whole percent.

1. before: $24
after: $20

2. before: 4,985 people
after: 5,500 people

3. before: 15.6 mL
after: 20.2 mL

4. before: 190 pounds
after: 150 pounds

5. before: 130 minutes
after: 150 minutes

6. before: 92 liters
after: 110 liters

7. before: $98
after: $75

8. before: 2,850 points
after: 1,775 points

9. Explain how to find a percent of change.

10. Most cars depreciate once they've been bought and are driven off a dealer's lot. Mike bought a car in 2000 for $19,500. The value of his car in 2002 was $12,225. What was the percent of decrease of the value of the car?

Name Date

Comparing rational numbers

Rational Numbers

A rational number is any number that can be named by a fraction with a numerator and a denominator that are integers. For example, $-\frac{1}{2}$, $4\frac{3}{4}$, and -3.25 are each rational numbers because $-\frac{1}{2} = \frac{-1}{2}$, $4\frac{3}{4} = \frac{19}{4}$, and $-3.25 = -3\frac{1}{4} = -\frac{13}{4} = \frac{-13}{4}$.

To compare two rational numbers, simply rewrite each so that they have the same positive denominator and then compare the numerators.

Compare $-\frac{8}{9}$ and $\frac{1}{3}$.

$-\frac{8}{9} = \frac{-8}{9}$

$\frac{3}{4} = \frac{3}{9}$ Change to the same denominator.

$-8 < 3$ Compare the numerators.

So $\frac{-8}{9} < \frac{3}{9}$ and $-\frac{8}{9} < \frac{1}{3}$.

If two fractions have the same positive denominator, the fraction with the greater numerator is the greater number.

Compare $-\frac{5}{7}$ and $-\frac{6}{7}$

$-5 > -6$

So $-\frac{5}{7} > -\frac{6}{7}$.

Circle the fraction that is greater in each set.

1. $\frac{6}{13}$ or $\frac{8}{13}$

2. $-\frac{1}{3}$ or $\frac{1}{3}$

3. $-\frac{2}{9}$ or $-\frac{7}{9}$

4. $-\frac{9}{20}$ or $-\frac{13}{20}$

5. $\frac{5}{12}$ or $-\frac{11}{12}$

6. $-\frac{1}{3}$ or $\frac{3}{4}$

Write <, >, or = in each ☐ to make a true sentence.

7. $\frac{7}{10}$ ☐ $\frac{3}{10}$

8. $\frac{5}{16}$ ☐ $\frac{3}{4}$

9. $\frac{1}{2}$ ☐ $\frac{1}{4}$

10. $-\frac{7}{8}$ ☐ $-\frac{8}{9}$

11. $-\frac{11}{24}$ ☐ $-\frac{5}{8}$

12. $\frac{7}{5}$ ☐ $\frac{6}{4}$

13. $-\frac{8}{12}$ ☐ $-\frac{4}{6}$

14. $-\frac{3}{4}$ ☐ $-\frac{9}{12}$

15. $\frac{2}{3}$ ☐ $-\frac{3}{4}$

16. $-\frac{7}{4}$ ☐ $-\frac{5}{3}$