

Adding integers**Integer Exploration**

Two situations are possible when adding integers: adding integers with same signs and adding integers that have different signs. Remember these two rules when faced with these situations:

1. To add integers that have the same sign, simply add their absolute values. Give the result the same sign as the integers.

$$3 + 5$$

$$|3| + |5|$$

$$8$$

Answer is positive since the integers were both positive.

$$-4 + (-7)$$

$$|-4| + |-7|$$

$$4 + 7$$

$$-11$$

Answer is negative since the integers were both negative.

2. To add integers that have different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.

$$-7 + 5$$

$$|-7| - |5|$$

$$7 - 5$$

Subtract absolute values.

$$-2$$

Answer is negative because -7 has the greater absolute value.

State whether each sum is **positive**, **negative**, or **zero**.

1. $-3 + 5$

2. $16 + (-16)$

3. $25 + 45$

4. $-150 + 125$

5. $4 + (-11)$

6. $-11 + (-12)$

Find each sum.

7. $17 + (-6)$

8. $-8 + 3$

9. $9 + (-4)$

10. $-13 + (-10)$

11. $-12 + (-8)$

12. $-5 + (-15)$

13. Write an addition sentence for the following situation, then solve. Bob made a profit of \$4,500 last year and had a loss of \$4,800 this year.



Subtracting integers

Integer Exploration

Every integer has an opposite. An integer and its opposite are called additive inverses of each other. For example, 3 has an opposite of -3. If 3 and -3 are added together, their sum is 0. The sum of any integer and its opposite will always have a result of 0.

$$3 + (-3) = 0$$

$$-8 + 8 = 0$$

To subtract an integer, simply add its opposite. Thus, change the subtraction problem to an addition problem and solve using the rules already learned for addition of integers.

Subtract $10 - 12$

$$10 + (-12)$$

Change to addition by adding the opposite of 12.

$$|-12| - |10|$$

Subtract absolute values.

$$12 - 10$$

$$-2$$

Give result a negative sign since -12 has the greater absolute value.

Subtract $6 - (-13)$

$$6 + 13$$

Change to addition by adding the opposite of -13.

$$19$$

Result is positive since adding numbers with the same positive sign.

Change each problem to an addition problem. Tell whether each answer will be **positive**, **negative**, or **zero**.

1. $1 - 3$

2. $10 - (-5)$

3. $-8 - 14$

4. $-4 - 7$

5. $-2 - (-12)$

6. $20 - 18$

Solve each expression.

7. $-10 - 6$

8. $12 - (-9)$

9. $-13 - (-8)$

10. $7 - 16$

11. $24 - (-26)$

12. $-5 - 15$

13. $-40 - 3$

14. $23 - 30$



15. Explain how the subtraction of integers is related to the addition of integers.

Multiplying integers

Integer Exploration

Multiplying integers is just like multiplying positive whole numbers except there is a possibility of negative numbers. Remember the following rules when multiplying integers:

1. The product of two integers with the same sign is positive.

Multiply 12×2

24 Result is a positive 24 since both integers were positive.

Multiply $-4 \cdot (-9)$

36 Result is a positive 36 since both integers were negative.

2. The product of two integers with different signs is negative.

Multiply -5×7

-35 Result is a negative 35 since the integers had different signs.

Look at the following example with multiplying 3 integers.

Multiply $-3(-4)(-5)$

12(-5)

-60

Multiply -3 and -4 and the result is a positive 12.

Result is negative since multiplying integers with different signs.

State whether each product is **positive**, **negative**, or **zero**.

1. 7×4

2. $-3 \times (-5)$

3. -7×0

4. $6 \times (-8)$

5. $-9 \cdot 9$

6. $-2 \times (-11)$

Solve each expression.

7. -10×7

8. $-16 \cdot (-2)$

9. $-12 \times (-11)$

10. -15×0

11. 5×8

12. $8 \times (-10)$

13. $6 \times (-20)$

14. $-3 \cdot 13$

15. Multiply $(-2)(-3)(4)$. Multiply $(-5)(-6)(-1)$.

Looking at the number of negative signs in both problems, write a rule that will help determine the sign of the product if multiplying two or more integers.



Dividing integers

Integer Exploration

When dividing integers, it is important to remember the following rules:

1. When dividing two integers with the same sign, the quotient is positive.

Divide $-10 \div (-5)$

2

Result is a positive 2 since both integers had the same sign.

Divide $\frac{60}{5}$

12

Result is a positive 12 since both integers had the same sign.

2. When dividing two integers with different signs, the quotient is negative.

Divide $-45 \div 9$

-5

Result is a negative 5 since integers had different signs.

Note: The rules are the same for multiplication of integers as they are for the division of integers.

State whether each quotient is **positive**, **negative**, or **zero**.

1. $9 \div 3$

2. $45 \div (-15)$

3. $-28 \div 4$

4. $\frac{0}{-7}$

5. $-16 \div (-8)$

6. $-26 \div (-13)$



Find the value of each expression.

7. $-121 \div (-11)$

8. $\frac{0}{15}$

9. $100 \div (-20)$

10. $\frac{20}{-5}$

11. $-9 \div 3$

12. $\frac{-38}{-19}$

13. $96 \div 16$

14. $\frac{-21}{-7}$

15. Describe a real-life situation in which negative numbers are used. Write your own word problem involving these negative numbers and division.

Solving inequalities by adding and subtracting

Integer Exploration

To solve an inequality that involves addition, solve it using subtraction just as you would when solving equations. Likewise, use addition to solve an inequality that involves subtraction. Be sure to always check each solution.

Solve $a + 5 > 11$

$a + 5 - 5 > 11 - 5$

$a > 6$

Subtract 5 from each side of the inequality.

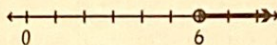
Solve for the variable.

Check: Choose a value greater than 6 and substitute it into the original inequality.

$15 + 5 > 11$

$20 > 11$

True sentence



Solve $b - 2 \leq 3$

$b - 2 + 2 \leq 3 + 2$

$b \leq 5$

Note: \leq means less than ($<$) or equal to ($=$).

Add 2 to each side of the inequality.

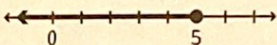
Solve for the variable.

Check: Choose a value less than or equal to 5 and substitute it into the original inequality.

$4 - 2 \leq 3$

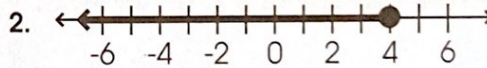
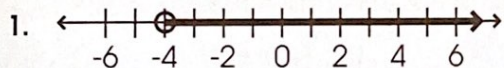
$2 \leq 3$

True sentence



To graph each of these solutions, simply draw a number line and draw a dot on the number found in the solution. The dot will be filled in if the inequality is \leq or \geq . The dot will be left unfilled if the inequality is $<$ or $>$. Then simply draw an arrow to the right if it is greater than and to the left if it is less than. See the above graphs.

Write an inequality for each solution set graphed below.



Solve each inequality and check the solution.

3. $x - 7 < 15$

4. $-7 + a < 4$

5. $r \leq -15 - 9$

6. $n - 15 \geq -12$

7. $j + 10 \geq 22$

8. $e - 10 > 5$

9. $y + 8 > -12$

10. $-30 < x - 6$

Write an inequality and graph the solution.

11. Barbara's class can have no more than 30 kids.

12. The Schmidt's house is more than 150 years old.

Solving inequalities by multiplying and dividing

Integer Exploration

When solving inequalities that involve multiplication or division, it is important to remember the two properties of inequality:

1. When multiplying or dividing each side of an inequality by a positive integer, the inequality symbol stays the same.

Solve $3x > 30$

$$\frac{3x}{3} > \frac{30}{3}$$

$$x > 10$$

Divide each side by 3.

Solve for x .

2. When multiplying or dividing each side of an inequality by a negative integer, the inequality symbol must be reversed.

Solve $\frac{x}{-4} \leq 8$

$$\frac{x}{-4} \cdot -4 \leq 8 \cdot -4$$

$$x \geq -32$$

Multiply each side by -4 .

Solve for x and reverse the inequality symbol since multiplying by a negative number.

Note: It is always a good idea to check each solution by putting it back into the original inequality and making sure it creates a true sentence.

1. Write an inequality that can be solved using multiplication where the solution is $x < 12$.
2. Write an inequality that can be solved using division where the solution is $x > 3$.

Solve each inequality and check the solution.

3. $-5y < -35$

4. $7a \leq -84$

5. $\frac{m}{3} \leq -10$

6. $-121 \geq -11z$

7. $\frac{x}{-4} \geq 21$

8. $-72 > 4b$

9. $-12t > 108$

10. $\frac{x}{-2} < -8$

11. The product of an integer and 5 is greater than -32 . Find the least integer that makes this true.
12. The quotient of an integer and -3 is less than -18 . Find the least integer that makes this true.

Multiplying and dividing monomials

Properties Used in Algebra

Remember the parts that make up a power. For example, 4^2 is a power, where 4 is the base and 2 is the exponent. In mathematics, powers that have the same base can be multiplied simply by adding their exponents.

$$4^5 \cdot 4^3 = 4^{5+3} = 4^8 \quad \text{The base is 4 and remains unchanged. Add the exponents.}$$

$$n^2 \cdot n^7 = n^{2+7} = n^9 \quad \text{The base is } n \text{ and remains unchanged. Add the exponents.}$$

Find the product of $(4x^5)(-3x^7)$.

$$\begin{aligned} & -12x^{5+7} && \text{Multiply 4 and -3. The base } x \text{ remains unchanged.} \\ & = -12x^{12} && \text{Add the exponents.} \end{aligned}$$

Powers that have the same base can also be divided by simply subtracting the exponents.

$$\frac{10^4}{10^2} = 10^{4-2} = 10^2 \quad \text{The base is 10. Subtract exponents.}$$

$$\text{Divide } \frac{-9ab^7}{3b^5}$$

$$= -3ab^{7-5} \quad \text{Divide -9 and 3.}$$

$$= -3ab^2 \quad \text{Subtract exponents of base } b.$$

- Write a division and a multiplication problem, each with a solution of 5^4 .
- Explain the relationship between multiplying and dividing powers.

Find each product or quotient. Leave each answer in exponential form.

3. $11^6 \cdot 11^7$

4. $b^9 \cdot b^4$

5. $\frac{z^9y^5}{z^3y^3}$

6. $x^3 \cdot x^8$

7. $\frac{3^7}{3^2}$

8. $\frac{-12x^{12}}{3x^7}$

9. $4^5 \cdot 4^2$

10. $\frac{10^{12}}{10^8}$

11. $4c^4d \cdot -5cd^3$

Find each missing exponent.

12. $(6^?) (6^4) = 6^{12}$

13. $\frac{r^?}{r^5} = r^8$

14. $a(a^3)(a^5) = a^?$

15. $\frac{13^?}{13^?} = 1$

Negative exponents

Properties Used in Algebra

Any negative exponent can be made positive by simply moving the exponent from the numerator to the denominator, or vice versa.

Write each expression using positive exponents.

1. 12^{-4}

$$= \frac{1}{12^4} \quad \text{Move the base 12 and its exponent to the denominator and the exponent becomes positive.}$$

2. xy^{-5}

$$= \frac{x}{y^5} \quad \text{Move the base } y \text{ and its exponent to the denominator. } x \text{ remains in the numerator because it already has a positive exponent.}$$

Write each expression using negative exponents.

1. $\frac{1}{6^8}$

$$= 6^{-8} \quad \text{Move the base 6 and its exponent to the numerator.}$$

2. $\frac{4}{5^6}$

$$= 4 \cdot 5^{-6} \quad \text{Move the base 5 and its exponent to the numerator.}$$

Find the product of $(b^3)(b^{-7})$ and express using positive exponents.

$$(b^3)(b^{-7}) = b^{3+(-7)} = b^{-4} = \frac{1}{b^4} \quad \text{Move } b \text{ and its negative exponent to the denominator.}$$

Write each expression using positive exponents.

1. 4^{-2}

2. 7^{-3}

3. b^{-12}

4. xy^{-5}

5. x^{-7}

6. 5^{-10}

7. $15^{-4}f^{-1}$

8. $3(ab)^{-6}$

Write each fraction as an expression using negative exponents.

9. $\frac{1}{6^4}$

10. $\frac{1}{18}$

11. $\frac{2}{4^3}$

12. $\frac{c}{d^5}$

13. $\frac{1}{b}$

14. $\frac{1}{y^7}$

15. $\frac{x}{y^8}$

16. $\frac{4a}{2b^3}$

Find the product or quotient using positive exponents in the answer.

17. $(a^{-7})(a^3)$

18. $\frac{c^2}{c^5}$

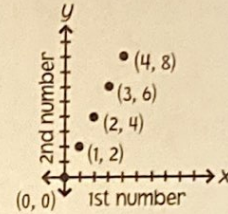
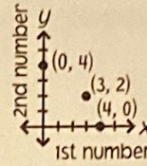
Ordered pairs and graphing

An ordered pair is a pair of numbers written in a specified order. For example, $(3, 2)$ is an ordered pair. Any ordered pair of numbers can be graphed. For instance, to graph the ordered pair $(3, 2)$, you must move 3 units to the right and then up 2 units.

On the same graph, graph the ordered pairs $(4, 0)$ and $(0, 4)$. To graph $(4, 0)$, move 4 units to the right and up 0 units. To graph $(0, 4)$, move 0 units to the right and up 4 units. Graph each of the ordered pairs in the table.

x (1st number)	0	1	2	3	4
y (2nd number)	0	2	4	6	8

Note: The x -value is always the first number in the ordered pair and moves to the right and the y -value is the second number in the ordered pair and moves up.



State the moves that would be made to graph each ordered pair.

1. $(4, 2)$

2. $(7, 10)$

3. $(2, 2)$

4. $(3, 0)$

5. $(3, 11)$

6. $(0, 5)$

Name the ordered pair for each point.

7. A

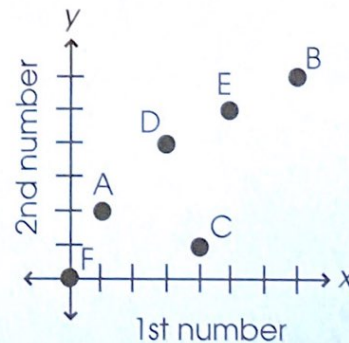
8. B

9. C

10. D

11. E

12. F



Graph the set of ordered pairs by drawing your own graph.

13. $(5, 1)$, $(0, 2)$, $(2, 4)$, $(1, 0)$, $(6, 4)$, $(3, 2)$

Integers and the coordinate system

Graphing

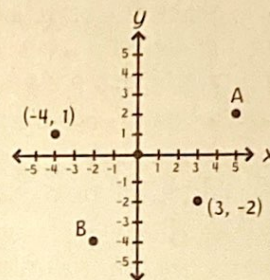
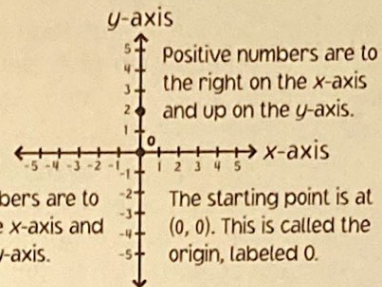
Integers are graphed on a number line. Ordered pairs of integers are graphed on two number lines, called axes. The horizontal number line is called the x -axis, and the vertical number line is called the y -axis. Look at the coordinate system to the right with important parts labeled.

To graph an ordered pair of integers, starting at the origin, move right (x is positive) or left (x is negative), then up (y is positive) or down (y is negative). For example, graph $(3, -2)$ and $(-4, 1)$ on the graph below.

The numbers of an ordered pair are called coordinates. To name the coordinates of a point, state the ordered pair of numbers that corresponds to the point. For example, on the graph, find the coordinates of A and B.

The coordinates of A: $(5, 2)$.

The coordinates of B: $(-2, -4)$.



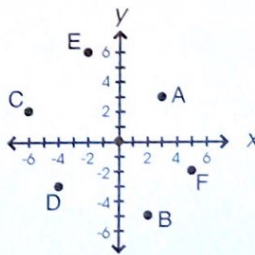
- Which direction are the positive numbers on the x -axis?
- Which direction are the negative numbers on the y -axis?

State the moves that would be made to graph each ordered pair.

- | | | |
|--------------|---------------|--------------|
| 3. $(3, -2)$ | 4. $(7, 6)$ | 5. $(0, -3)$ |
| 6. $(-1, 4)$ | 7. $(-4, -5)$ | 8. $(7, 0)$ |

Name the coordinates of each point.

- | | |
|-------|-------|
| 9. A | 10. B |
| 11. C | 12. D |
| 13. E | 14. F |



- On graph paper, draw and label a pair of axes. Then graph each point below, labeling each point with its letter.

A $(-1, 5)$, B $(3, -4)$, C $(0, -2)$, D $(6, 3)$, E $(-4, -1)$, F $(-5, 0)$