

Name

Date

**Order of operations**

Connecting with Algebra

To simplify an expression, use the order of operations to calculate the answer. The order of operations is as follows:

1. parentheses ( ) and brackets [ ], called grouping symbols (worked from the inside out)
2. exponents
3. multiplication and division (worked from left to right)
4. addition and subtraction (worked from left to right)

$5 + 12 \div 4$

$6[10 - (2 + 8)]$

$4 - 3y, y = 2$

$7a - 4b, a = 3, b = 2$

$5 + 3$

$6[10 - 10]$

$4 - 3 \cdot 2$

$7 \cdot 3 - 4 \cdot 2$

$8$

$6 \cdot 0$

$4 - 6$

$21 - 18$

$0$

$3$

$3$

State the first operation to be performed in each expression.

1.  $4 \cdot 7 - 5$

2.  $(10 + 4) - 2 \cdot 5$

3.  $7 - 2 + 5 - 1$

4.  $8 - 18 \div 6$

5.  $9 + 8 \div 2$

6.  $18 \div 9 + 3$

7.  $9 \cdot 6 \div 9$

8.  $2(4(12 - 7) + 5)$

Simplify.

9.  $14 + 7 \cdot 3$

10.  $2(5 - (3 + 1) + 13)$

11.  $33 - 4(15 \div 3)$

12.  $10 \div 2 - 3$

13.  $24 - (18 + 4 - 10 \cdot 2)$

14.  $19 + 3((8 + 3) - 9 \div 3)$

Evaluate.

15.  $14 \cdot 4 \div r, r = 2$

16.  $7a - 3 \cdot 5, a = 5$

17.  $22 - 22 \div s, s = 2$

18.  $18 - 3x, x = 4$

19.  $16 + y \div 8, y = 48$

20.  $12x + 7, x = 5$

## Exponents and powers

Connecting with Algebra

Exponents are used to represent repeated multiplication. An exponent represents the number of times the base is used as a factor. For example,  $3^4$  is an expression used to represent 3 that is a factor 4 times. The number 3 is the base, the number 4 is the exponent, and  $3^4$  is the power.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$$

Evaluate  $x^3$  when  $x = 4$

$$(2x)^2, x = 3$$

$$3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 3^2 5^4$$

$$x^3 = 4 \cdot 4 \cdot 4$$

$$(2x)^2 = (2 \cdot 3)^2$$

$$= 64$$

$$= 36$$

It is important to remember the order of operations when evaluating expressions that involve exponents. Remember: parentheses, exponents, multiplication/division, and addition/subtraction.

Write each expression in exponent form.

1.  $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

2.  $a \cdot a \cdot b \cdot b \cdot b$

3.  $9 \cdot 9$

4.  $x \cdot x \cdot x \cdot y$

5.  $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

6.  $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

Simplify.

7.  $2^3$

8.  $(35 - 5)^2 + 15$

9.  $6^2$

10.  $35 - 5^2$

11.  $7^2 \cdot 2$

12.  $6^2 - 3 \cdot 5$

13.  $5^3 \cdot 3$

14.  $(8 + 2)^3$

Evaluate.

15.  $4x^2 + 2x, x = 4$

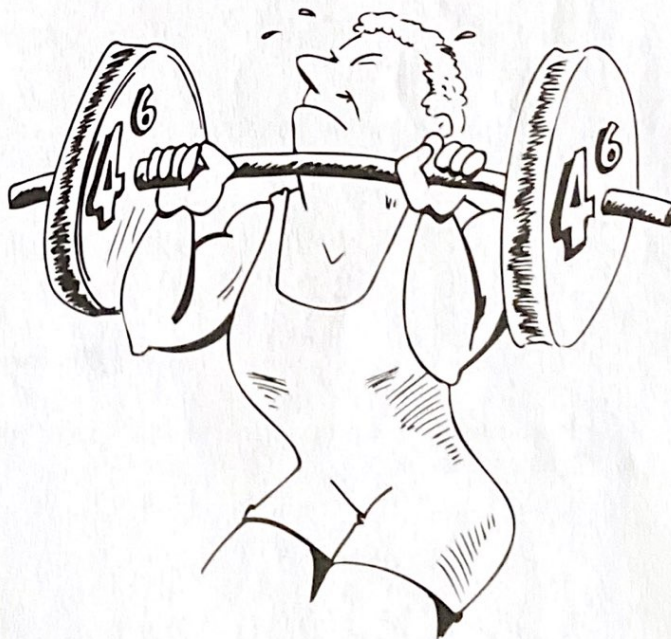
16.  $6r^2 + r, r = 5$

17.  $k^2 - 2k + 12, k = 4$

18.  $t^2 + 3t - 4, t = 3$

19.  $(2y)^3 + 2y, y = 5$

20.  $24 - 10s + 6s^2, s = 2$



**Geometric formulas****Connecting with Algebra**

Just as variables are used in algebra, they are also used in geometric situations. Geometric formulas often use letters to represent the sides of geometric shapes. The **perimeter** of a figure is the total distance around. The **area** of a figure is the number of square units it contains. The perimeter and area of geometric figures can be represented with formulas.

Perimeter of a square =  $4s$ , where  $s$  is the length of one side

Perimeter of a rectangle =  $2\ell + 2w$ , where  $\ell$  is the length and  $w$  is the width

Area of a square =  $s^2$ , where  $s$  is the length of one side

Area of a rectangle =  $\ell w$ , where  $\ell$  is the length and  $w$  is the width

Area of a triangle =  $\frac{1}{2}bh$ , where  $b$  is the base and  $h$  is the height

Look at these examples.

The area of a square with a side length of 4 inches:

$$A = s^2 = 4^2 = 4 \cdot 4 = 16 \text{ in.}^2$$

The perimeter of a rectangle with a length of 10 cm and a width of 3 cm:

$$P = 2\ell + 2w = 2 \cdot 10 + 2 \cdot 3 = 20 + 6 = 26 \text{ cm}$$

Use  $P = 4s$  to find the perimeter of each square.

1.  $s = 12 \text{ ft.}$

2.  $s = 20 \text{ cm}$

3.  $s = 7 \text{ in.}$

4.  $s = 18 \text{ mm}$

Use  $P = 2\ell + 2w$  to find the perimeter of each rectangle.

5.  $\ell = 13 \text{ in.}, w = 5 \text{ in.}$

6.  $\ell = 2 \text{ cm}, w = 20 \text{ cm}$



First identify the formula to use from the letters given, then use that formula to find the area of the figure given its dimensions.

7.  $b = 7 \text{ in.}, h = 4 \text{ in.}$

8.  $s = 13 \text{ mm}$

9.  $\ell = 2.1 \text{ yd.}, w = 7 \text{ yd.}$

10.  $s = 6 \text{ cm}$

11.  $b = 20 \text{ ft.}, h = 5 \text{ ft.}$

12.  $s = 3.4 \text{ in.}$

13.  $\ell = 15 \text{ ft.}, w = 3 \text{ ft.}$

14.  $\ell = 24 \text{ cm}, w = 8 \text{ cm}$

15.  $b = 30 \text{ mm}, h = 2 \text{ mm}$

## Combination of like terms

### Connecting with Algebra

The expression  $3x^2 + 2x + 1$  has three terms,  $3x^2$ ,  $2x$ , and  $1$ . The definition of terms in an expression is those parts of the expression connected by addition. A term in an expression without a variable is called a constant, as  $1$  is above. For terms to be considered "like" terms, they must have the same variable and corresponding variables must have the same exponents. All constant terms are considered "like" terms.

like terms

$3x$  and  $8x$

$2x^2y$  and  $3x^2y$

unlike terms

$4y$  and  $10z$

$3ab$  and  $4ac$

In the example,  $3x$  and  $8x$  are like terms with numerical coefficients of  $3$  and  $8$ . A numerical coefficient of a term is simply the number before its corresponding variable. When combining like terms, simply keep the variable the same and combine the numerical coefficients.

$$4y + 10y = 14y$$

$$10x - 3x = 8x$$

$$6b + 4b - 5b = 10b$$

$$12x^2y - 10x^2y + 2x^2y = 4x^2y$$

Identify the like terms in each problem.

1.  $7c + 12c - 2$

2.  $19y - 10$

3.  $12rt - 10r + 18t$

4.  $5r - 10r + 8rs$

5.  $5t + 7t - 1$

6.  $q + 9 + 2q + 5q$

Simplify. If not possible, write **already simplified**.

7.  $8m - 3m$

8.  $8y + 12y + 3y$

9.  $3s + 4(7s - 2)$

10.  $2 + 10k$

11.  $8q + 10q + 14$

12.  $4 + 8x + 11y$

13.  $5a + 6a - 9a$

14.  $t + 8m + 4t - 4m$

15.  $4(5w + 2) + w$

Simplify. Then evaluate given the value of the variable.

16.  $6(3a + 4) + 5(4a - 2)$ ,  $a = 5$

17.  $5(b + 7) + 2b - 14 + (b + 10)$ ,  $b = 8$

**Solution sets of sentences**

Connecting with Algebra

Equations and inequalities are considered mathematical sentences. An equation is simply two expressions connected together by an equal sign. An inequality is two expressions with an inequality symbol, such as  $>$ ,  $<$ , or  $\neq$  between them. Mathematical sentences can be either true or false.

$$9(5 - 4) = 10$$

$$9(1) = 9 \neq 10, \text{ so false}$$

$$27 - (5 + 20) = 2$$

$$27 - 25 = 2 = 2, \text{ so true}$$

Mathematical sentences can also be called open sentences. This is when the equation or inequality has a variable, such as  $3x + 4 = 10$  or  $2x - 10 > 15$ . An open sentence is neither true nor false until the variable is replaced with numbers that make the sentence true. These numbers that can replace the variable are called the replacement set of numbers for the variable. The solution set of numbers for an open sentence is simply the numbers that are part of the replacement set that make the sentence true.

For example, with a replacement set of  $\{2, 4, 6\}$ ,  $3x + 4 = 10$  has a solution set of  $\{2\}$  because  $3 \cdot 2 + 4 = 10$  is a true sentence. With the replacement set of  $\{10, 12, 14\}$ , the inequality  $2x - 10 > 15$  has a solution set of  $\{14\}$  because  $2 \cdot 14 - 10 > 15$  is a true sentence. If no number in the replacement set makes the sentence true, then the solution set of the sentence is simply the empty set, or the null set, written  $\emptyset$ .

Write **true** or **false**.

1.  $7 > 4$

2.  $9 + 11 < 21$

3.  $8 > 4 + 5$

4.  $2(6 + 8) \neq 3 \cdot 9$

Write **yes** or **no** to tell whether each number is in the solution set for the open sentence.

5.  $14 = x + 2$ ; 12

6.  $6.7 \neq b$ ; 6.7

7.  $2x - 10 < 14$ ; 10

8.  $14 - x \geq 7$ ; 7

9.  $15 - y > 9$ ; 6

10.  $x + 15 = 20$ ; 5

11.  $a^2 > a$ ; 2

12.  $4c - 4 < 15$ ; 5

Find the solution set for each problem if the replacement set is  $\{0, 1, 2, 3, 4\}$ .

13.  $5x - 10 < 8$

14.  $3x + 6 = 4x + 4$

15.  $4x + 10 \neq x + 9$

Find the solution set for each problem if the replacement set is  $\{2, 4, 6, 8\}$ .

16.  $7x - 2 = 8x - 8$

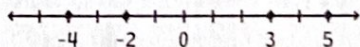
17.  $2y + 3 \leq 3y + 2$

18.  $5z + 10 > 4z + 12$

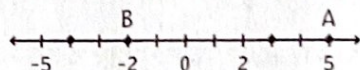
## The real number line

Using Rules of Algebra

Real numbers are all of the numbers that are used in algebra. These numbers can be pictured as points on a horizontal line called a real number line. A starting point for all number lines is the origin, which is the point 0. The numbers to the left of 0 are the negative numbers. The numbers to the right of 0 are the positive numbers. Zero is neither positive nor negative. Below is an example of a number line with -4 and -2 as the negative numbers and 3 and 5 as the positive numbers.

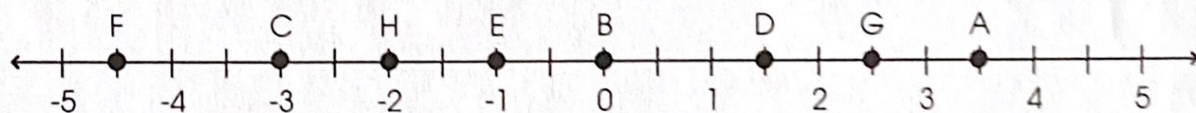


When graphing numbers on a line, the point is called the graph of the number, and the number that corresponds to the point is called the coordinate of the point.



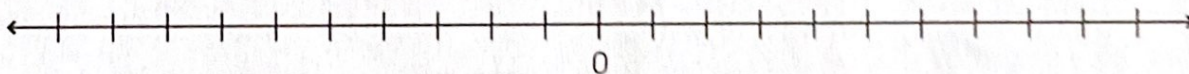
In the above graph, point A is the graph of the number 5. The number 5 is the coordinate of point A. Point B is the graph of the number -2. The number -2 is the coordinate of the point B.

Give the coordinate of each point graphed.



1. A      2. B      3. C      4. D      5. E      6. F      7. G      8. H

On the number line, graph each point whose coordinate is given.



9. A: -2      10. B:  $3\frac{1}{2}$       11. C:  $-4\frac{1}{2}$       12. D: 0      13. E:  $-1\frac{1}{2}$       14. F: 5

Write each set of numbers in increasing order.

15. 2.1, -1.8, 3, 0,  $\frac{1}{3}$ ,  $-\frac{1}{3}$

16. 7.5,  $-\frac{1}{2}$ , 7, -5.4,  $\frac{3}{4}$ ,  $\frac{1}{2}$

17. 18, -15, 12, 3, -13, -11

18. -6, -8, 13, 4, 5, -10

Use < or > to make each statement true.

19. 8 \_\_\_\_\_ 4

20. -3 \_\_\_\_\_ 1

21.  $2\frac{1}{2}$  \_\_\_\_\_ 3

22.  $-3\frac{1}{3}$  \_\_\_\_\_ 3

23. 0 \_\_\_\_\_ -4

24.  $4\frac{1}{4}$  \_\_\_\_\_ 4

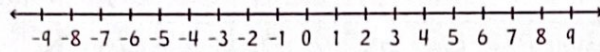
25. -9 \_\_\_\_\_ -6

26. 2 \_\_\_\_\_ -3

**Addition of real numbers**

Using Rules of Algebra

A number line is a great way to model the addition of real numbers.

Add  $3 + 5$ . (Start at 3 and move 5 places to the right since 5 is positive.) The answer is 8.Add  $3 + (-5)$ . (Start at 3 and move 5 places to the left since 5 is negative.) The answer is -2.

When adding a number that is positive, move to the right. When adding a number that is negative, move to the left.

Explain how to add the following numbers using a number line, and tell the result.

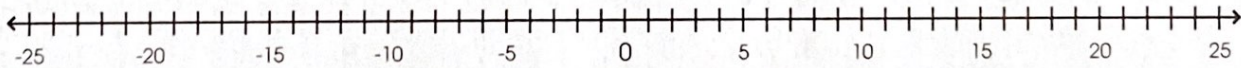
1.  $7 + 8$  \_\_\_\_\_

2.  $-5 + 9$  \_\_\_\_\_

3.  $10 + (-7)$  \_\_\_\_\_

4.  $-11 + (-6)$  \_\_\_\_\_

Use the number line to add the numbers.



5.  $4 + 9$

6.  $-5 + 13 + (-11)$

7.  $-7 + (-6)$

8.  $14 + 10 + (-3)$

9.  $0 + (-5)$

10.  $-6 + (-9) + 10$

11.  $-3 + 3$

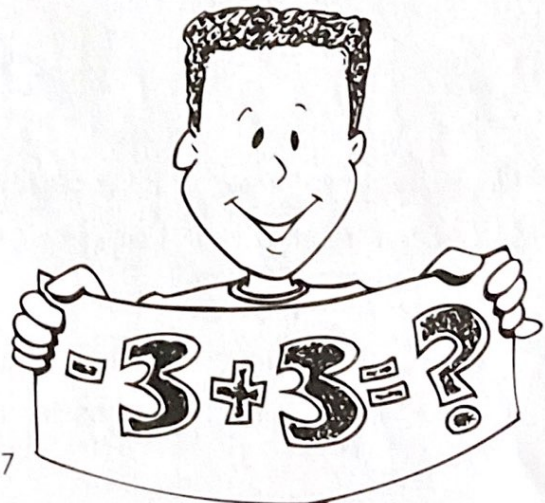
12.  $8 + (-2) + (-4) + 6$

13.  $7 + (-12)$

14.  $-11 + 5 + (-13) + 8$

15.  $-11 + 8 + (-1)$

16.  $12 + (-6) + (-14) + 17$



**Addition of real numbers**

Using Rules of Algebra

To add two real numbers with the same sign:

1. Add their absolute values.
2. Determine the sign of the answer.
  - a. If both numbers are positive, then the answer is positive.
  - b. If both numbers are negative, then the answer is negative.

$$-2 + (-3) = -5$$

$$4 + 5 = 9$$

$$-10 + (-21) = -31$$

To add two real numbers with different signs, if the numbers are not opposites:

1. Subtract their absolute values, the larger number minus the smaller number.
2. The sign of the answer will be the same sign of the number with the larger absolute value.

$$3 + (-10)$$

$$|-10| - |3|$$

$$10 - 3 = 7$$

$$\text{answer: } -7$$

$$-12 + 4$$

$$|-12| - |4|$$

$$12 - 4 = 8$$

$$\text{answer: } -8$$

$$-9 + 13$$

$$|13| - |-9|$$

$$13 - 9 = 4$$

$$\text{answer: } 4$$

Add.

1.  $4 + 3$

2.  $-12 + 4$

3.  $19 + (-3) + 6$

4.  $11 + 12$

5.  $-21 + 0$

6.  $-1 + (-4) + 18$

7.  $-2 + 5$

8.  $23 + (-15)$

9.  $-7 + 8 + (-5)$

10.  $7 + (-8)$

11.  $-11 + 11$

12.  $32 + (-15) + (-6)$

13.  $-10 + (-15)$

14.  $-22 + (-22)$

15.  $13 + (-21) + 7$

Write an expression to represent each situation and solve.

16. A football team had a 5-yard loss followed by an 8-yard gain. Find the resulting gain or loss.

17. In one month, Jeff lost 8 pounds. The next month he gained 5 pounds. He lost 4 more pounds in the third month. Find the net gain or loss.



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**Subtraction of real numbers**

Using Rules of Algebra

For all real numbers  $a$  and  $b$ ,  $a - b = a + (-b)$ . Simply stated: To subtract a number, add its opposite.

$$\begin{aligned} 7 - 10 &= 7 + (-10) \\ &= -3 \end{aligned}$$

$$\begin{aligned} 5 - 8 + 3 - 1 &= 5 + (-8) + 3 + (-1) \\ &= 5 + 3 + (-8) + (-1) \text{ (group the + and} \\ &= 8 + (-9) \text{ - numbers)} \\ &= -1 \end{aligned}$$

$$\begin{aligned} -4 - (-12) \\ &= -4 + 12 \\ &= 8 \end{aligned}$$

Change each problem into an addition problem.

1.  $7 - 9$

2.  $-6 - (-4)$

3.  $-11 - 5$

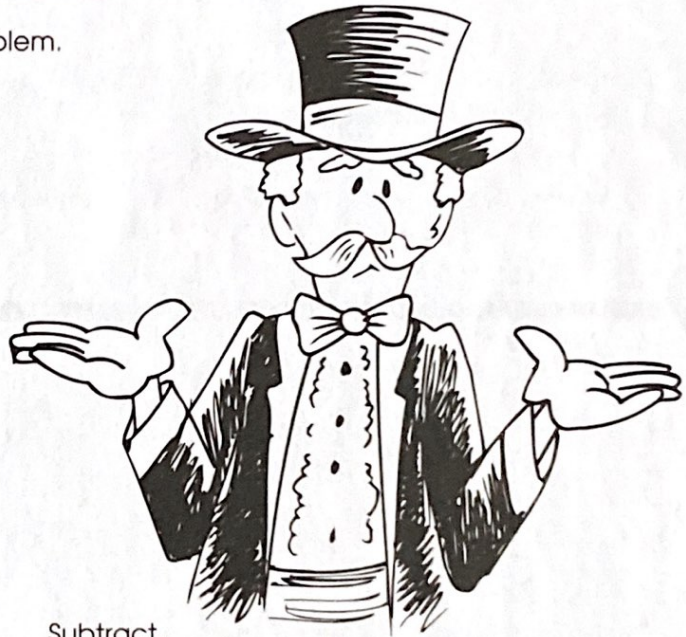
4.  $12 - (-15)$

5.  $8 - 3$

6.  $22 - 5$

7.  $4 - 11$

8.  $-4 - (-9)$



Subtract.

9.  $9 - 11$

10.  $0 - (-12)$

11.  $-5 - 4$

12.  $6 - (-6)$

13.  $-1 - (-1)$

14.  $-7 - 6$

15.  $3 - (-5)$

16.  $17 - 23$



Evaluate when  $x = -2$ ,  $y = -5$ , and  $z = 12$ .

17.  $x - y$

18.  $y - x$

19.  $x - z$

20.  $z - y$

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## Multiplication of real numbers

Using Rules of Algebra

The property of zero for multiplication: For all real numbers  $a$ ,  $a \cdot 0 = 0$  and  $0 \cdot a = 0$ .

Simply stated, any real number multiplied by 0 is 0. For example,  $0 \cdot 20 = 0$  and  $13 \cdot 0 = 0$

To multiply two real numbers with same signs:

1. Multiply their absolute values.
2. The sign of their product is positive.

$$\begin{array}{ccccc} \text{positive} & \cdot & \text{positive} & = & \text{positive} \\ (+) & & (+) & & (+) \end{array}$$

$$3 \cdot 12 = 36$$

$$\begin{array}{ccccc} \text{negative} & \cdot & \text{negative} & = & \text{positive} \\ (-) & & (-) & & (+) \end{array}$$

$$-7 \cdot -8 = 56$$

To multiply two real numbers with opposite signs:

1. Multiply their absolute values.
2. The sign of their product is negative.

$$\begin{array}{ccccc} \text{negative} & \cdot & \text{positive} & = & \text{negative} \\ (-) & & (+) & & (-) \end{array}$$

$$-2 \cdot 5 = -10$$

$$\begin{array}{ccccc} \text{positive} & \cdot & \text{negative} & = & \text{negative} \\ (+) & & (-) & & (-) \end{array}$$

$$4 \cdot -8 = -32$$

Write the sign of the product for each number.

1.  $(-10)4$

2.  $8(-1)$

3.  $(-2)(-3)$

4.  $(7)(5)(-3)$

5.  $5(6)$

6.  $(-2)(-7)$

7.  $(-12)(-4)(-1)$

8.  $(-6)(4)(-2)$

Multiply to find each product.

9.  $4(6)(-1)$

10.  $(-1)(-4)(-3)$

11.  $(-\frac{1}{2})(2)$

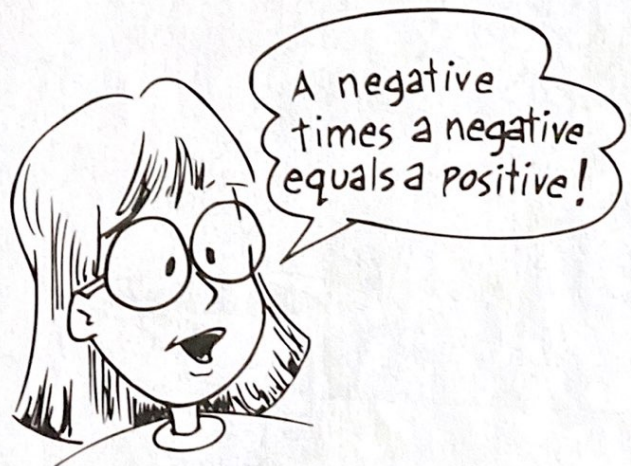
12.  $(7)(-3)(0)$

13.  $(5)(3)$

14.  $(-\frac{1}{8})(-16)(4)$

15.  $(-9)(-4)$

16.  $(-7)(7)$



Evaluate when  $x = -3$ ,  $y = -5$ , and  $z = 0$ .

17.  $xy$

18.  $-3yz$

19.  $xy^2$

20.  $4x^2y^2z^2$