Mixed operations

When simplifying problems using all the rules for operating with real numbers, be very careful not to confuse the rules.

$$-2 + (-8) = -10 \qquad -2 - (-8) = -2 + 8 \qquad -2 \bullet -8 = 16 \qquad -2 \div (-8) = -2 \bullet -\frac{1}{8}$$
$$= 6 \qquad\qquad\qquad\qquad = \frac{1}{4}$$

Remember:
1. The sum of two negative numbers is negative.
2. The difference between two negative numbers has the sign of the number with the greater absolute value.
3. The product of two negative numbers is positive.
4. The quotient of two negative numbers is positive.

Simplify.

1. $-3 + 4 \bullet -9$

2. $6 - 8^2$

3. $6^2 - (-1)^2$

4. $-4 \bullet 7 - 5$

5. $-2 + (-10)^2$

6. $11 - (-3)^3$

Evaluate each problem given each variable value.

7. $x^3 - 5, x = -4$

8. $-8 - 7t^2, t = -3$

9. $8 - 4y, y = 6$

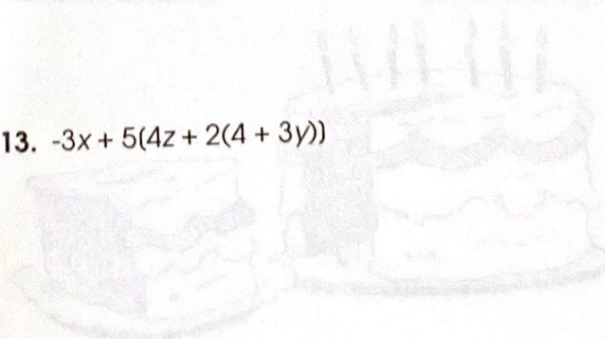**10.** $\frac{a + 6b}{-2a}, a = -2, b = 5$

Simplify. Then evaluate when $x = -2$, $y = -3$, and $z = 4$.

11. $3(4x + 2(8y - 7))$

12. $6x \div (2(6 + 4y)) - 3$

13. $-3x + 5(4z + 2(4 + 3y))$

14. $(2y - 3z) \div (-2x + 5)$

Algebra

Like terms with real number coefficients

The property of -1 for multiplication states that for any real number a, $-1a = -a$ and $a(-1) = -a$.

$-a + 3a = (-1 + 3)a = 2a$ (The variable a has a coefficient -1. It is simply not written.)

$a + 3a = (1 + 3)a = 4a$ (The variable a has a coefficient 1. It is simply not written.)

To combine like terms with real number coefficients, use the rules of adding, subtracting, multiplying, and dividing real numbers and simplify.

$$-6ab - 7ab = (-6 - 7)ab \qquad 10y - 13 - 12y + 4 = (10 - 12)y + (-13 + 4)$$
$$= [-6 + (-7)]ab \qquad\qquad\qquad = [10 + (-12)]y + (-9)$$
$$= -13ab \qquad\qquad\qquad\qquad = -2y + (-9) = -2y - 9$$

Simplify.

1. $8y + 10y$

2. $-7b - 11b$

3. $-4r - t + 4r + 7t$

4. $-6x + 8x$
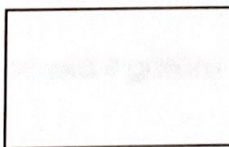
5. $-4c - c$

6. $-8d + 7w + (-8d) + 10w$

7. $3a - 5a$
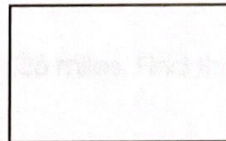
8. $6s - 6s$

9. $-4a + 5b - a - 4b$

Write a formula for the perimeter of each rectangle. Simplify. Remember: The basic formula for the perimeter of a rectangle is $P = 2\ell + 2w$.
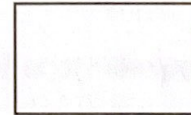
10. $3x$

$x + y$

11. $3y + 4x$

$x - 2y$

12. $6x - y + 3$

$5x + 3y - 4$

negative factors

The distributive property states: $a(b + c) = ab + ac$, where a is multiplied by both b and c. When a is negative, a is a negative factor. Example, $-a(b + c) = -ab + (-ac)$, where the negative factor a is being distributed to both b and c making both products negative.

$$-3(2x + 9) = -3 \cdot 2x + (-3) \cdot 9$$
$$= -6x + (-27)$$
$$= -6x - 27$$

$$3x - (8 + 6x) = 3x - 1(8 + 6x)$$
$$= 3x + (-1)8 + (-1)6x$$
$$= 3x + (-8) + (-6x)$$
$$= -3x - 8$$

Simplify.

1. $-8(3x + 2)$

2. $-6(4 + 2y) - 9$

3. $t - (7 - t)$

4. $-4(-8 + 4y)$

5. $3s - (4s + 5)$

6. $6 - (8 - b) - 4b$

7. $-(-a + b - 2)$

8. $10 - 8(5 - 6d) + 12d$

9. $-5(3z - 6) - 2(7 + 3z)$

Simplify. Then evaluate each problem given each variable value.

10. $-(-8x + 3) - 9x + 2, x = 4$

11. $-(6z - 9) - (10 - z), z = -2$

12. $-9y - (6y + 5) + 11y, y = -1$

13. $7a - (9 - a) + 15, a = -5$

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Name _____   Date _____

Topics covered:

The Real Number Line   Division of Real Numbers
Opposites (Additive Inverses)   Mixed Operations
  and Absolute Value   Like Terms with Real Number Coefficients
Addition of Real Numbers   Negative Factors
Subtraction of Real Numbers   Rates and Ratios
Multiplication of Real Numbers

Graph each point on the number line.

$$0$$
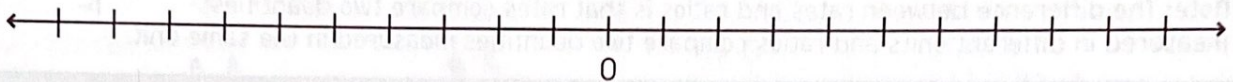
1. A: $-1\frac{1}{2}$ **2.** B: 2   **3.** C: -4   **4.** D: $3\frac{1}{2}$ **5.** E: 0

Write a real number to represent each situation. Then give its opposite.

6. a withdrawal of $12.96   **7.** a gain of 13 yards   **8.** a deposit of $145.50

Find the absolute value.

9. $|-11|$ **10.** $|13|$ **11.** $-|3 \cdot 5|$ **12.** $|-2 + (-5)|$ **13.** $-|-10|$ **14.** $|2.1|$

Simplify.

15. $-3 + 5$ **16.** $8 - 10$ **17.** $(-\frac{1}{3})(3)$ **18.** $-55 \div 11$

19. $-2 + 7 + (-6)$ **20.** $-3 - (-11)$ **21.** $(-2)(-7)(-1)$ **22.** $\frac{-7}{-7}$

Simplify using mixed operations.

23. $7^2 - (-1)^2$ **24.** $-2 + 4 \cdot -12$ **25.** $9 - 6^2$ **26.** $(6 + (-2) - 3^2) \div -5$

Simplify by combining like terms.

27. $-7x + 9x$ **28.** $-3x - 5y + 2x + 10y$ **29.** $-4(5x - 11) - (6 + 2x)$

30. You drove 256 miles in 4 hours. What was your average speed?

Solving equations using multiple steps

When solving equations for a particular variable, sometimes you need to use more than one of the properties of equality.

$3x - 2 = 7$

$3x - 2 + 2 = 7 + 2$ Add 2 to both sides of the equation.

$\dfrac{3x}{3} = \dfrac{9}{3}$ Now, divide both sides of the equation by 3.

$x = 3$ Check $3(3) - 2 = 7$

                              $9 - 2 = 7$

                              $7 = 7$ true statement

Thus, the solution is 3.

Here are some steps to follow when solving multi-step equations.

1. Simplify both sides of the equation (if needed).
2. Use the addition or subtraction property of equality to isolate terms containing the variable.
3. Use the multiplication or the division property of equality to further isolate the variable.
4. Check the solution.

Solve each equation for *x*. Check your answers.

1. $6x - 3 = 21$ **2.** $6 + \dfrac{x}{4} = -1$ **3.** $18 - 3x = -12$

4. $7 + 2x = -13$ **5.** $-4 = 7x + 8 - 8x$ **6.** $13 = 9 - \dfrac{x}{5}$

7. $-7 - x = -5$ **8.** $5x + 9 - 4x = 12$ **9.** $-8x - 13 = 19$

10. $-3 = -5 - 2x$ **11.** $\dfrac{1}{3}x + 9 = 15$ **12.** $-7 = 3x - 15 - 7x$

Translate each problem into an equation. Solve each equation.

13. Thirty-two is 7 less than 3 times a number. Find the number.

14. Negative twenty-five is 4 times a number increased by 7. Find the number.

15. Twelve pounds less than twice Jane's weight is 270 pounds. What is Jane's weight?

Name _____   Date _____

Solving equations with variables on both sides

Often, equations with a variable on both sides of the equation need to be solved. This requires one additional step of getting the variable on one side only.

$4x - 1 = 2x + 7$

$4x - 2x - 1 = 2x - 2x + 7$ Get the variable on one side of the equation.

$2x - 1 = 7$ Now, solve for x using the properties of equality.

$2x - 1 + 1 = 7 + 1$

$\dfrac{2x}{2} = \dfrac{8}{2}$

$x = 4$

Check $4(4) - 1 = 2(4) + 7$

$16 - 1 = 8 + 7$

$15 = 15$ true statement

Thus, the solution is 4.

note: Combine the variables on the side of the equation with the greater variable coefficient, in order to avoid solving an equation with a variable which has a negative coefficient.

Solve each equation for x. Check your answers.

1. $9x - 12 = 3x$

2. $8x - 12 = 15x - 4x$

3. $11 + 6x = 2x - 13$

4. $-5x = 9 - 2x$

5. $-8x - 10 = 4x + 14$

6. $10x - 5 = 21 - 3x$

7. $-12x = 14 - 5x$

8. $19 - 3x = 21 + x$

9. $4x + 12 = -3x - 6 + 4x$

Translate each problem into an equation. Solve each equation.

10. Twice Beth's daily pay is the same as her daily pay increased by $45. Find Beth's daily pay.

11. A number decreased by 12 is the same as 3 times the number. Find the number.

12. The temperature, increased by 65°, is the same as 5 times the temperature decreased by 15°. Find the temperature.

Solving equations with parentheses

Equations often contain parentheses. When asked to solve equations such as these, first use the distributive property and then combine like terms, solving the rest of the equation using the properties of equality.

$6x - 4(2 - 3x) = 28$ Use the distributive property to begin simplifying.
$6x - 8 + 12x = 28$ Combine like terms.
$18x - 8 = 28$ Solve equation for x.
$18x - 8 + 8 = 28 + 8$
$\frac{18x}{18} = \frac{36}{18}$
$x = 2$

Check $6(2) - 4[2 - 3(2)] = 28$
$12 - 4(2 - 6) = 28$
$12 + 16 = 28$
$28 = 28$ true statement

Thus, the solution is 2.

Use the distributive property to remove the parentheses.

1. $3(5x - 10)$

2. $(6x + 5)(-2)$

3. $-4(x - 6) = 2(7 - 7x)$

4. $-5(2 - 5x)$

5. $(-12 - 7x)(4)$

6. $8(-9x + 4) = -3(6x + 9)$

Solve each equation for x. Check your answers.

7. $3(x + 7) = 30$

8. $2(x + 3) = 12 - x$

9. $-(x + 7) - 5 = 4(x + 3) - 6x$

10. $-5(x + 4) = 20$

11. $5(5 - x) = 4(x - 5)$

12. $5(x - 1) = 2x + 4(x - 1)$

Translate each problem into an equation. Solve each equation.

13. Four times the sum of a number and 7 is 44 less than the number. Find the number.

14. Sixteen more than a number is the same as 8 times the sum of the number and 9.

Slope of a line

The slope of a line is the number of units a line rises or falls for each unit of horizontal change from left to right on its graph. The slope of a line is represented by the letter m and can be found given two points (x_1, y_1) and (x_2, y_2) on the line using the following formula.

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

If the slope is greater than 0 (which is positive), then the line rises from left to right.
If the slope is less than 0 (which is negative), then the line falls from left to right.
If the slope is equal to 0, then the line is horizontal.
If the slope is undefined, then the line is vertical.

Find the slope of the line passing through the points (-5, 6) and (-3, 10).

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 6}{-3 - (-5)} = \frac{4}{-3 + 5} = \frac{4}{2} = \frac{2}{1} = 2$$

Thus, since the slope is a positive 2, then we know the line rises from left to right.

Determine whether the line **rises from left to right**, **falls from left to right**, is **vertical**, or is **horizontal**, given the following slopes.

1. $m = 2$

2. $m = -5$

3. $m = 6$

4. $m = $ undefined

5. $m = 0$

6. $m = -7$

Find the slope of the line that passes through each set of points.

7. (5, 4), (6, 9)

8. (-3, 4), (-1, 2)

9. (-6, -3), (-2, 9)

10. (-2, -1), (0, 3)

11. (6, -5), (3, 10)

12. (7, 8), (1, -16)

Sketch the line by plotting the points. Find the slope of the line passing through the points.

13. (0, 0), (2, 4)

14. (-1, -2), (-3, 6)

15. (4, -5), (5, -10)

16. (-2, 0), (-5, 6)

MINUTE 1

1. $2^3 =$

2. $27 \div 9 + 3 =$

3. If $m + 40 = 75$, then $m =$ _____.

4. Number of letters in the alphabet minus the
   number of months in a year? _____

5. $(4 + 2)^2 =$

6. Write $3 \cdot 3 \cdot 3 \cdot 3$ in exponential form. _____

7. $8 \cdot 9 =$

8. $\dfrac{48}{6} =$

9. $1^{10} =$

10. $5 + (4)(3) =$

BONUS! Farmer Doug has some pigs and chickens.
One day he counted 24 legs and 7 heads in the barnyard.
How many of each animal did Farmer Doug count? _____

MINUTE 2

1. $(2)(3)(4) =$

2. Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ in exponential form. _____

3. $\dfrac{4+6}{5} =$

4. Bobby thinks that $5^2 = 10$.
What is wrong with this answer? _____

5. $4 + 6 \cdot 2 = 4 + 12$ Circle: True   or   False

6. If $a = 5$ and $b = 6$, then what does ab equal? _____

7. Miss White wants to buy 5 value meals at Mel's Diner.
What is a reasonable total for her purchase?
    a. $25   **b.** $1,000   **c.** $100   **d.** $10

8. 12 snakes have how many eyes altogether? _____

9. $5 + (9)(6) =$

10. Which of these operations should be completed first
when solving an equation?
    a. $\times$ **b.** $+$ **c.** $(\ )$ **d.** $\div$