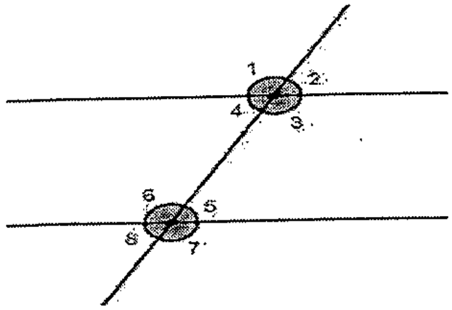
5/13

TOPIC 2 PARALLEL & PERPENDICULAR LINES

2-1: Parallel Lines – lines that never intersect. A line that goes through other lines is called the transversal.



Corresponding Angles – angles in the same spot on the other parallel line. They are congruent to each other. The corresponding angles in the picture are: $\measuredangle 1 \cong \measuredangle 6, \measuredangle 2 \cong \measuredangle 5, \measuredangle 4 \cong \measuredangle 8, \measuredangle 3 \cong \measuredangle 7$

Alternate Interior Angles – angles between the parallel lines & on opposite sides of the transversal but not touching each other. They are congruent too. $\measuredangle 3 \cong \measuredangle 6$ & $\measuredangle 4 \cong \measuredangle 5$

Alternate Exterior Angles – angles on the outside of the parallel lines and on opposite sides of the transversal but not touching each other. They are congruent as well. $\measuredangle 1 \cong \measuredangle 7$ & $\measuredangle 2 \cong \measuredangle 8$

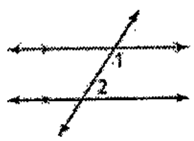
Same Side Interior Angles – angles between the parallel lines and on the same side of the transversal. They are supplementary, add to 180. $m\measuredangle 4 + m\measuredangle 5 = 180°$ & $m\measuredangle 3 + m\measuredangle 5 = 180°$

Same Side Exterior Angles – angles on the outside of the parallel lines and on the same side of the transversal. They are supplementary, add to 180. $m\measuredangle 1 + m\measuredangle 8 = 180°$ & $m\measuredangle 2 + m\measuredangle 7 = 180°$

There are four special angle relationships formed when parallel lines are intersected by a transversal.

POSTULATE 2-1 Same-Side Interior Angles Postulate

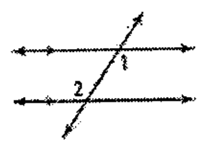
If...



Then... $m\angle 1 + m\angle 2 = 180°$

THEOREM 2-1 Alternate Interior Angles Theorem

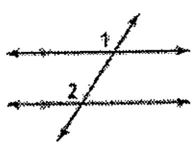
If...



Then... $\angle 1 \cong \angle 2$

THEOREM 2-2 Corresponding Angles Theorem

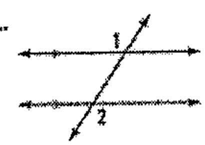
If...



Then... $\angle 1 \cong \angle 2$

THEOREM 2-3 Alternate Exterior Angles Theorem

If...



Then... $\angle 1 \cong \angle 2$

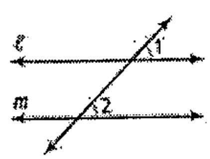
2-2: Proving Lines Parallel

Converse of the Corresponding Angles Theorem

If two lines and a transversal form corresponding angles that are congruent, then the lines are parallel.

PROOF: SEE EXERCISE 8.

If...



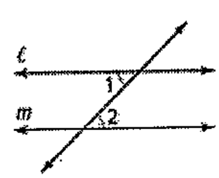
Then... $\ell \parallel m$

Converse of the Alternate Interior Angles Theorem

If two lines and a transversal form alternate interior angles that are congruent, then the lines are parallel.

PROOF: SEE EXAMPLE 2.

If...

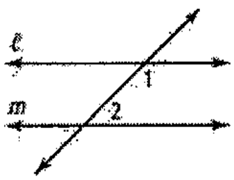


Then... $\ell \parallel m$

Converse of the Same-Side Interior Angles

If two lines and a transversal form same-side interior angles that are supplementary, then the lines are parallel.

If... $m\angle 1 + m\angle 2 = 180$



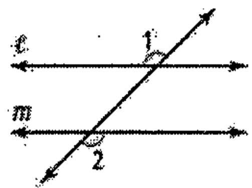
Then... $\ell \parallel m$

THEOREM 2-8

Converse of the Alternate Exterior Angles

If two lines and a transversal form alternate exterior angles that are congruent, then the lines are parallel.

If...



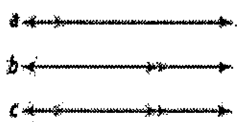
Then... $\ell \parallel m$

THEOREM 2-9

If two lines are parallel to the same line, then they are parallel to each other.

PROOF: SEE EXERCISE 17.

If...

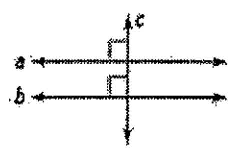


Then... $a \parallel b$

If two lines are perpendicular to the same line, then they are parallel to each other.

PROOF: SEE EXERCISE 18.

If...



Then... $a \parallel b$

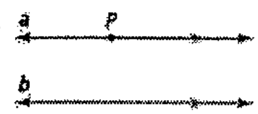
2-3 Parallel Lines & Triangle Angle Sums

THEOREM 2-10

Through a point not on a line, there is one and only one line parallel to the given line.

PROOF: SEE EXERCISE 10.

If...



Then... line a is the only line parallel to line b through P.

Angle Measures of Triangles

WORDS

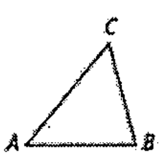
Interior Angle Measures

The sum of the measures of all the angles of a triangle is 180°.

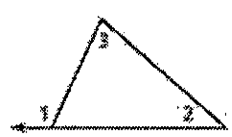
Exterior Angle Measure

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

DIAGRAM



$m\angle A + m\angle B + m\angle C = 180°$



$m\angle 1 = m\angle 2 + m\angle 3$

5/13

2-4: Slopes of Parallel Lines & Perpendicular Lines

Slope is represented with the letter m. $m = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x} = \frac{y_1 - y_2}{x_1 - x_2}$

Parallel Slopes are equal so Ex. $m_1 = -2$ & $m_2 = -2$ or $y = -1/_2 x + 3$ & $y = -1/_2 x - 2$

Perpendicular Slopes are Opposite Reciprocals & multiply to -1 so $m = -\frac{1}{m}$

Ex. . $m_1 = -2$ & $m_2 = \frac{1}{2}$ or $y = -1/_2 x + 3$ & $y = 2x - 2$

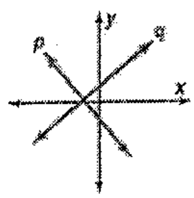
THEOREM 2-14

Two non-vertical lines are perpendicular if and only if the product of their slopes is −1.

A vertical line and a horizontal line are perpendicular to each other.

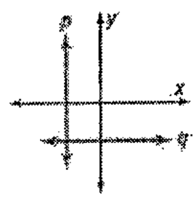
PROOF: SEE LESSON 7-4.

If... p and q are both not vertical



Then... $p \perp q$ if and only if the product of their slopes is −1

If... one of p and q is vertical and the other is horizontal



Then... $p \perp q$

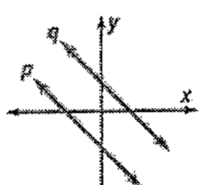
THEOREM 2-13

Two non-vertical lines are parallel if and only if their slopes are equal.

Any two vertical lines are parallel.

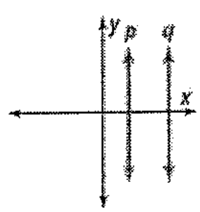
PROOF: SEE LESSON 7-5.

If... p and q are both not vertical



Then... $p \parallel q$ if and only if the slope of line p = slope of line q

If... p and q are both vertical



Then... $p \parallel q$

Slope – Intercept Form of a line is $y = mx + b$ *where m is slope & b is y – intercept.*

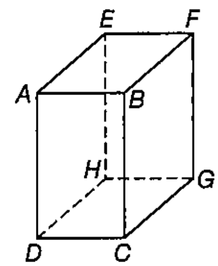
Point Slope Form; $y - y_1 = m(x - x_1)$ *where m is slope and* (x_1, y_1) *are coordinates of a point on the line.*

NAME ________________ DATE ________ PERIOD ________

Skills Practice

Parallel Lines and Transversals

For Exercises 1–4, refer to the figure at the right to identify each of the following.



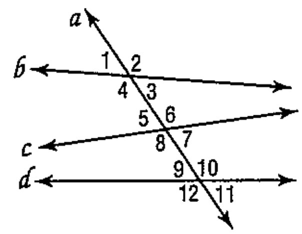
1. all planes that are parallel to plane DEH

2. all segments that are parallel to $\overline{AB}$

3. all segments that intersect $\overline{GH}$

4. all segments that are skew to $\overline{CD}$

Classify the relationship between each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.



5. $\angle 4$ and $\angle 5$ **6.** $\angle 5$ and $\angle 11$

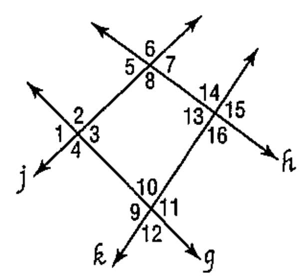
7. $\angle 4$ and $\angle 6$ **8.** $\angle 7$ and $\angle 9$

9. $\angle 2$ and $\angle 8$ **10.** $\angle 3$ and $\angle 6$

11. $\angle 1$ and $\angle 9$ **12.** $\angle 3$ and $\angle 9$

13. $\angle 6$ and $\angle 12$ **14.** $\angle 7$ and $\angle 11$

Identify the transversal connecting each pair of angles. Then classify the relationship between each pair of angles.



15. $\angle 4$ and $\angle 10$ **16.** $\angle 2$ and $\angle 12$

17. $\angle 7$ and $\angle 3$ **18.** $\angle 13$ and $\angle 10$

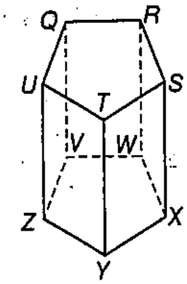
19. $\angle 8$ and $\angle 14$ **20.** $\angle 6$ and $\angle 14$

NAME ______________________ DATE __________ PERIOD ______ 5/13

Practice

Parallel Lines and Transversals

Refer to the figure at the right to identify each of the following.



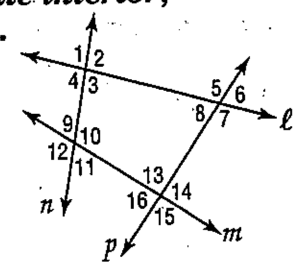
1. all planes that intersect plane STX

2. all segments that intersect $\overline{QU}$

3. all segments that are parallel to $\overline{XY}$

4. all segments that are skew to $\overline{VW}$

Classify the relationship between each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.



5. ∠2 and ∠10

6. ∠7 and ∠13

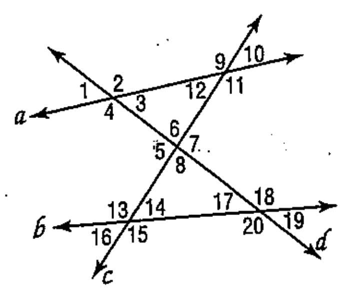
7. ∠9 and ∠13

8. ∠6 and ∠16

9. ∠3 and ∠10

10. ∠8 and ∠14

Name the transversal that forms each pair of angles. Then identify the special name for the angle pair.



11. ∠2 and ∠12

12. ∠6 and ∠18

13. ∠13 and ∠19

14. ∠11 and ∠7

FURNITURE For Exercises 15–16, refer to the drawing of the end table.

15. Find an example of parallel planes.

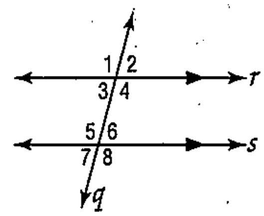
16. Find an example of parallel lines.

NAME ______________________ DATE ____________ PERIOD ____________

Skills Practice

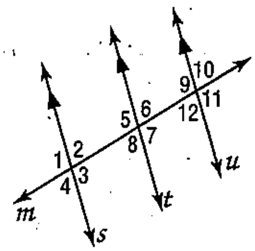
Angles and Parallel Lines

In the figure, $m\angle 2 = 70$. Find the measure of each angle.



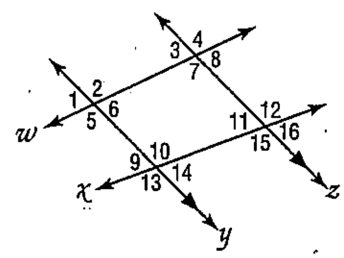
1. $\angle 3$
2. $\angle 5$
3. $\angle 8$
4. $\angle 1$
5. $\angle 4$
6. $\angle 6$

In the figure, $m\angle 7 = 100$. Find the measure of each angle.



7. $\angle 9$
8. $\angle 6$
9. $\angle 8$
10. $\angle 2$
11. $\angle 5$
12. $\angle 11$

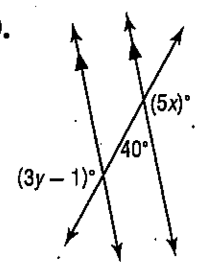
In the figure, $m\angle 3 = 75$ and $m\angle 10 = 105$. Find the measure of each angle.



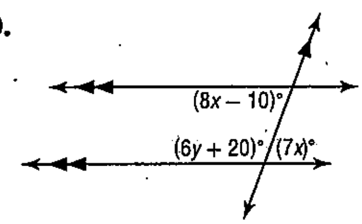
13. $\angle 2$
14. $\angle 5$
15. $\angle 7$
16. $\angle 15$
17. $\angle 14$
18. $\angle 9$

Find the value of the variable(s) in each figure. Explain your reasoning.

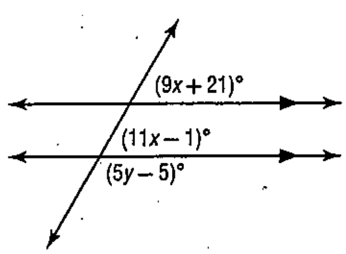
19.



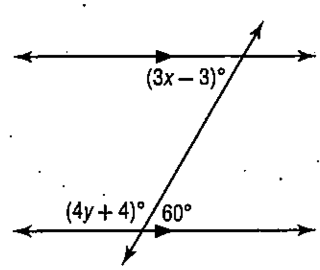
20.



21.



22.

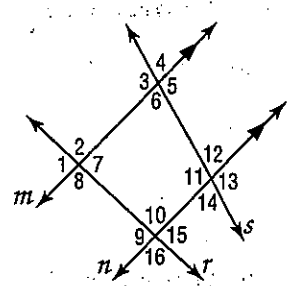


NAME ______________________ DATE ____________ PERIOD ____________ 5/13

Practice

Angles and Parallel Lines

In the figure, $m\angle 2 = 92$ and $m\angle 12 = 74$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.



1. $\angle 10$

2. $\angle 8$

3. $\angle 9$

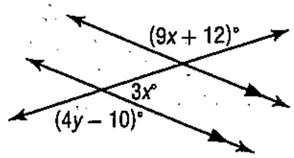
4. $\angle 5$

5. $\angle 11$

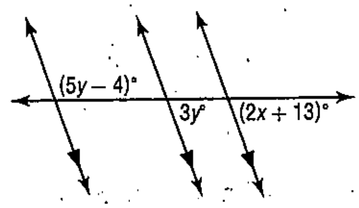
6. $\angle 13$

Find the value of the variable(s) in each figure. Explain your reasoning.

7.

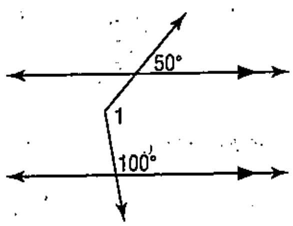


8.

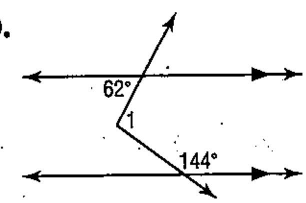


Find x. (*Hint*: Draw an auxiliary line.)

9.



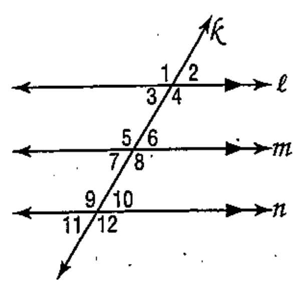
10.



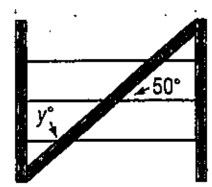
11. PROOF Write a paragraph proof of Theorem 3.3.

Given: $\ell \parallel m$, $m \parallel n$

Prove: $\angle 1 \cong \angle 12$



12. FENCING A diagonal brace strengthens the wire fence and prevents it from sagging. The brace makes a 50° angle with the wire as shown. Find the value of the variable.



NAME ________________ DATE ________ PERIOD ________

Skills Practice

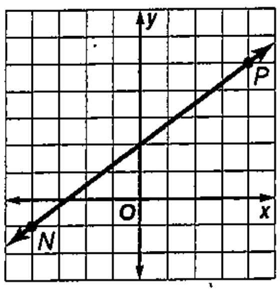
Slopes of Lines

Determine the slope of the line that contains the given points.

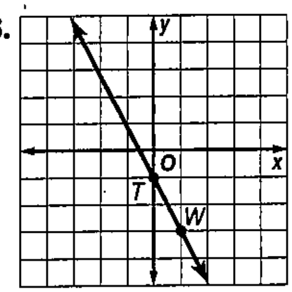
1. $S(-1, 2)$, $W(0, 4)$
2. $G(-2, 5)$, $H(1, -7)$
3. $C(0, 1)$, $D(3, 3)$
4. $J(-5, -2)$, $K(5, -4)$

Find the slope of each line.

5.



6.

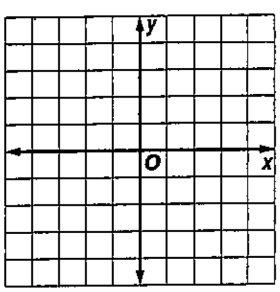


Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{MN}$ are *parallel*, *perpendicular*, or *neither*. Graph each line to verify your answer.

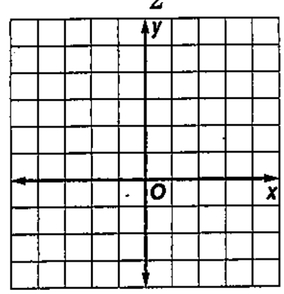
7. $A(0, 3)$, $B(5, -7)$, $M(-6, 7)$, $N(-2, -1)$
8. $A(-1, 4)$, $B(2, -5)$, $M(-3, 2)$, $N(3, 0)$
9. $A(-2, -7)$, $B(4, 2)$, $M(-2, 0)$, $N(2, 6)$
10. $A(-4, -8)$, $B(4, -6)$, $M(-3, 5)$, $N(-1, -3)$

Graph the line that satisfies each condition.

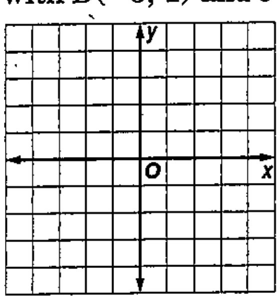
11. slope = 3, passes through $A(0, 1)$



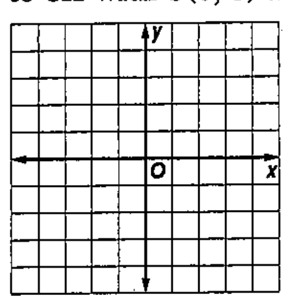
12. slope $= -\frac{3}{2}$, passes through $R(-4, 5)$



13. passes through $Y(3, 0)$, parallel to $\overleftrightarrow{DJ}$ with $D(-3, 1)$ and $J(3, 3)$



14. passes through $T(0, -2)$, perpendicular to $\overleftrightarrow{CX}$ with $C(0, 3)$ and $X(2, -1)$



Practice

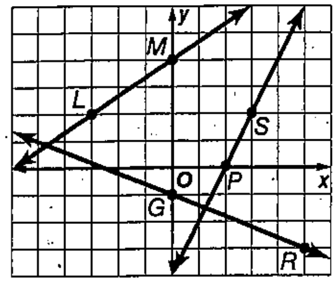
Slopes of Lines

Determine the slope of the line that contains the given points.

1. $B(-4, 4)$, $R(0, 2)$

2. $I(-2, -9)$, $P(2, 4)$

Find the slope of each line.



3. $\overleftrightarrow{LM}$

4. $\overleftrightarrow{GR}$

5. a line parallel to $\overleftrightarrow{GR}$

6. a line perpendicular to $\overleftrightarrow{PS}$

Determine whether $\overleftrightarrow{KM}$ and $\overleftrightarrow{ST}$ are *parallel, perpendicular*, or *neither*. Graph each line to verify your answer.

7. $K(-1, -8)$, $M(1, 6)$, $S(-2, -6)$, $T(2, 10)$

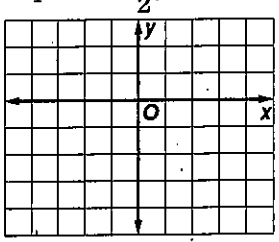
8. $K(-5, -2)$, $M(5, 4)$, $S(-3, 6)$, $T(3, -4)$

9. $K(-4, 10)$, $M(2, -8)$, $S(1, 2)$, $T(4, -7)$

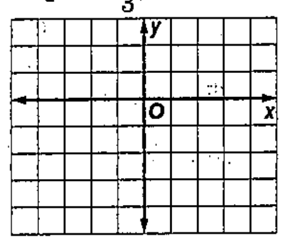
10. $K(-3, -7)$, $M(3, -3)$, $S(0, 4)$, $T(6, -5)$

Graph the line that satisfies each condition.

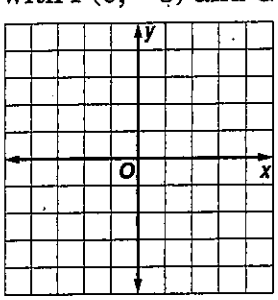
11. slope $= -\frac{1}{2}$, contains $U(2, -2)$



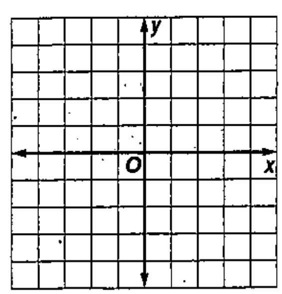
12. slope $= \frac{4}{3}$, contains $P(-3, -3)$



13. contains $B(-4, 2)$, parallel to $\overleftrightarrow{FG}$ with $F(0, -3)$ and $G(4, -2)$



14. contains $Z(-3, 0)$, perpendicular to $\overleftrightarrow{EK}$ with $E(-2, 4)$ and $K(2, -2)$



15. PROFITS After Take Two began renting DVDs at their video store, business soared. Between 2005 and 2010, profits increased at an average rate of \$9000 per year. Total profits in 2010 were \$45,000. If profits continue to increase at the same rate, what will the total profit be in 2014?

NAME ____________________ DATE __________ PERIOD __________

Skills Practice

Equations of Lines

Write an equation in slope-intercept form of the line having the given slope and y-intercept. Then graph the line.

1. m: −4, b: 3

2. m: 3, b: −8

3. m: $\frac{3}{7}$, (0, 1)

4. m: $-\frac{2}{5}$, (0, −6)

Write equations in point-slope form of the line having the given slope that contains the given point. Then graph the line.

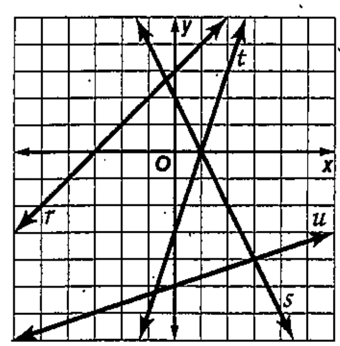
5. $m = 2$, (5, 2)

6. $m = -3$, (2, −4)

7. $m = -\frac{1}{2}$, (−2, 5)

8. $m = \frac{1}{3}$, (−3, −8)

Write an equation in slope-intercept form for each line shown or described.



9. r

10. s

11. t

12. u

13. the line parallel to line r that contains (1, −1)

14. the line perpendicular to line s that contains (0, 0)

15. $m = 6$, $b = -2$

16. $m = -\frac{5}{3}$, $b = 0$

17. $m = -1$, contains (0, −6)

18. $m = 4$, contains (2, 5)

19. contains (2, 0) and (0, 10)

20. x-intercept is −2, y-intercept is −1